# The numerical solution of inverse heat conduction problems with applications to reactor technology

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## 1. INTRODUCTION

Inverse heat conduction describes a physical situation where the temperature and heat flux at one surface of a domain have to be determined from corresponding measurements at the opposite side. The problem of inverse heat conduction always occurs when a surface may be unsuitable for attaching a sensor, or when the accuracy of a surface measurement may be seriously impaired by the presence of the sensor. Examples are present in every reactor pressure vessel, in the walls of reactor containments, or in piping systems; applications are also given in the lifetime evaluation of reactor components.

Even in its one-dimensional simplification, the underlying mathematical problem turns out to be an "illposed problem". This essentially means that small errors in the (measured) input data can lead to large errors in the temperatures and heat fluxes to be determined. The analysis of illposed problems is a growing field in the mathematical and engineering literature during the last years. However, a complete and rigorous treatment of inverse heat conduction problems has not yet been given. The latter is particularly true for corresponding numerical methods and their analysis. For two-dimensional problems, there seems to exist only one computer code (cf. Bass et al. (1980a)); however, results are not published in the open literature. For one-dimensional inverse heat conduction problems (abbreviation: IHCPs), there are several numerical methods among which those of Beck (1982a,1985a) are implemented in many codes. Numerical schemes solving IHCPs must be able to "stabilize" (or "regularize") the illposedness of the problems.

A computer code named SINCO (i.e. Finite Difference/Finite Element Solver for Inverse Heat Conduction) had been developed which solves one- and two-dimensional IHCPs. Moreover, a rigorous mathematical treatment of such problems together with an analysis of suitable numerical schemes had been initiated; a survey and first numerical results are published in (Baumeister & Reinhardt 1987a).

In this contribution, we state the problem, describe the numerical method, and present numerical results for a benchmark problem. Finally, post-testcalculations for HDR-fire-experiments will be shown.

## 2. STATEMENT OF THE PROBLEM AND NUMERICAL APPROXIMATION

The heat conduction equation for determing the temperature distribution is modelled by the following parabolic differential equation,

$$\rho\, c(u)\, \partial u/\partial t = \nabla \cdot \lambda(u)\, \nabla u$$

The heat conduction coefficient $\lambda$ and the specific heat capacity $c$ are allowed to be temperature dependent; $\rho$ denotes the density. Additionally an initial temperature distribution and boundary conditions for the domain under consideration have to be specified. We consider a situation

where three materials are present; Fig. 1 displays this situation and indicates the associated boundary conditions. Material 0 and 2 may be used for insulation purposes; they may also be absent. The problem consists in determining the temperature and flux at $x = 0$ in Material 1 from the corresponding quantities at $x = 1$. Such a problem is also called a "Cauchy problem". It is illposed in the sense described above. This can be rigorously shown by transforming the Cauchy problem into a family of Volterra integral equations of the first kind for determing the Fourier coefficients of the unknown temperature distribution at $x = 0$ (cf. (1987a)).

A first step in approximating the two-dimensional problem is performed when we discretize the y-interval into equidistant subintervals and approximate $\partial^2 u/\partial y^2$ by the second order central difference quotient. This leads to a coupled system of one-dimensional parabolic equations for the approximations $u_j$ of $u(.,y_j,t)$ on the j-th line, $y_j = j\,\Delta y$,

$$\frac{\partial u_j}{\partial t} = a\,\frac{\partial^2 u_j}{\partial x^2} + \frac{a}{(\Delta y)^2}\,(u_{j-1} - 2u_j + u_{j+1}), \qquad \begin{matrix} x \in [0,1],\ t > 0, \\ j=1,\ldots,J-1. \end{matrix}$$

Such an approximation is called a "line method approximation". Using the symmetry condition at $y = 0$ and the assumption of continuous flow from Material 1 to Material 2, the above system can be decoupled and leads to a (decoupled) system of one-dimensional IHCPs (cf. (1987a)). For one-dimensional problems, various numerical methods are available. In the program SINCO we use a Finite-Element modification of the Beck method (1985a).

Due to our specific decoupling procedure, it should be mentioned that our scheme can only take one "future time" into account. This means that we use the method of Stolz (1960) for the numerical solution of the decoupled system of one-dimensional IHCPs. Another way of decoupling, however, allows an arbitrary number of future times also for two-dimensional problems. It is well known that more future times make the numerical method more stable (while stepping in the time direction). The implementation of arbitrary future times into the computer code SINCO will be finished soon; details and numerical results will be published elsewhere.

## 3. NUMERICAL BENCHMARK TESTS

Computational results for a benchmark problem will be outlined in this section. More results are published in (1987a). For one-dimensional IHCPs, the "step heat flux test case" describes a situation where the heat flux is zero for $t < 0$ and is one for $t > 0$; the surface is insulated at $x = 1$. The associated temperature distribution is explicitly known (see $T^+(x,t)$ in (Beck et al. 1985a: Sec. 1.5)). With this test case, we construct a two-dimensional benchmark problem by extending the function constantly in the y-direction, $u(x,y,t) = T^+(x,t)$. At $y = d$, the temperature itself is given.

For our numerical calculations, we have set

$$\Delta t = 0.1,\ d_1 = 1,\ d = 2,\ J = 4,\ \Delta y = 0.25,\ \Delta x = 0.125 \text{ (equidistant)}.$$

The transducers are located at $x = 0.25$ and $x = 1.0$ for $j=0,1,2,3$. The temperature "data" at the transducer positions are computed from the truncated series of the exact solution (Case 1, in Fig. 2 to 4) and from perturbations of the form $\varepsilon \sin(\sigma t)$ of the latter (Case 2, in Fig. 4 only). In Fig. 2, the fluxes are shown at $x = 0$ and three different y-positions for Case 1; Fig. 3 presents the associated relative errors. The temperatures themselves at $x = 0$ are depicted graphically in Fig. 4 for both cases of temperature data where Case 2 with $\varepsilon = 0.01$ and $\sigma = 20$ is displayed only for $y = 0.5$. This demonstrates the stable behaviour of the method since perturbations in the data lead to perturbations in the numerical solutions of the same magnitude. All calculations are performed with one future time, as explained above. The agreement with the exact solution is satisfactory. More future times would presumably give more exact results. For other test cases, only one future time may cause an instable behaviour of the numerical approximations as demonstrated in (1987a).

## 4. NUMERICAL RESULTS FOR THE HDR-T51.16-EXPERIMENT

During the HDR-fire-experiments - details in Session J6 of this SMiRT-Conf., see (Müller 1987b) - there had been used Heat Transfer Measuring Blocks made of concrete for measuring the heat flow into walls and other materials inside a containment. These blocks are designed in such a way that a temperature profile inside the block is measured at several distinct points and thus allows to determine the heat flow coming into the front side of the block. Such a temperature profile - and a heat flow - arises since the containment atmosphere is heated during the fire and the block temperature follows with a certain delay.

The present test facilities allow the temperature measurement along the axis of the cylindric blocks. In the future, new blocks will be used measuring the temperature also in radial directions. The available experimental data can therefore be appropriately evaluated only by the one-dimensional version of the computer code SINCO. For the new blocks, the two-dimensional version will be used.

There already exists a one-dimensional program, called ALPHA, which in a semi-automatic way iteratively corrects the Alpha-number until the computed temperatures on the axis agree as good as possible with the measured ones. The Alpha-number is defined by the relation

$$q = \alpha(u_\infty - \vartheta)$$

where $q$ is the heat flux, $\vartheta$ the temperature at the surface and $u_\infty$ the ambient temperature.

Fig. 5 shows two calculated heat fluxes at the surface of a specific measuring block of the HDR-fire-experiment T51.16 computed by the program ALPHA, CQ2153, and by the 1-d. version of SINCO, CQ2001. In Fig. 6, the calculated temperatures at the surface CQ3153 and CQ1 coming out of ALPHA and SINCO, respectively, are displayed. The fluxes and temperatures agree very well.

Additionally, in Fig. 6 and Fig. 7, the measured temperatures CQ5320, CQ5321, CQ5322 and the corresponding numerical approximations CQ2, CQ3, CQ4 computed by SINCO (with 3 future times) at the locations 7.3, 19.3 and 30.6 mm away from the boundary are shown. Hereby, the ability of the method is demonstrated to fit the numerical solutions of the heat equation (in a least square sense) to the measured temperature data.

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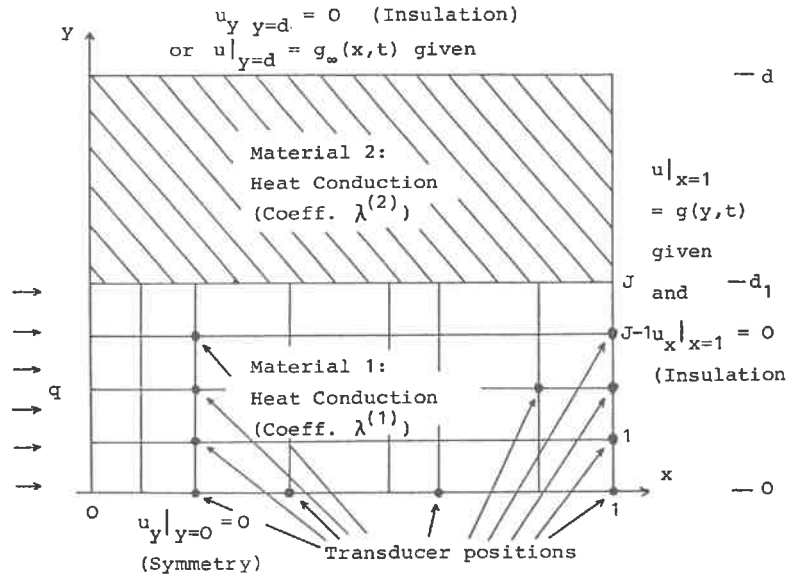


FIGURE 1 Inverse heat conduction problem at the boundary of a two-dimensional plate

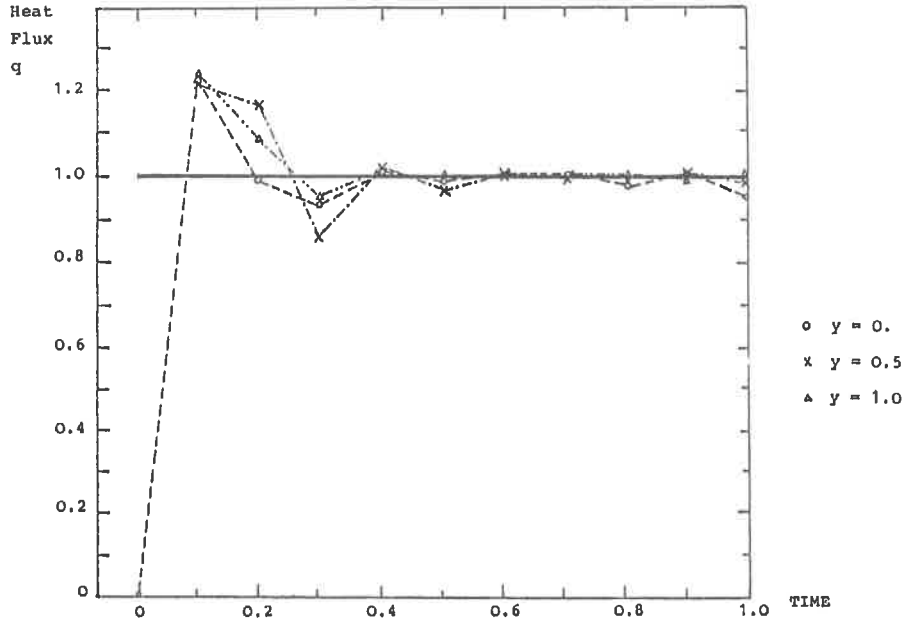


FIGURE 2 Calculated heat flux at x = 0 ; temperature data from truncated series (Case 1)
(Example : Step heat flux, constant in y-direction)

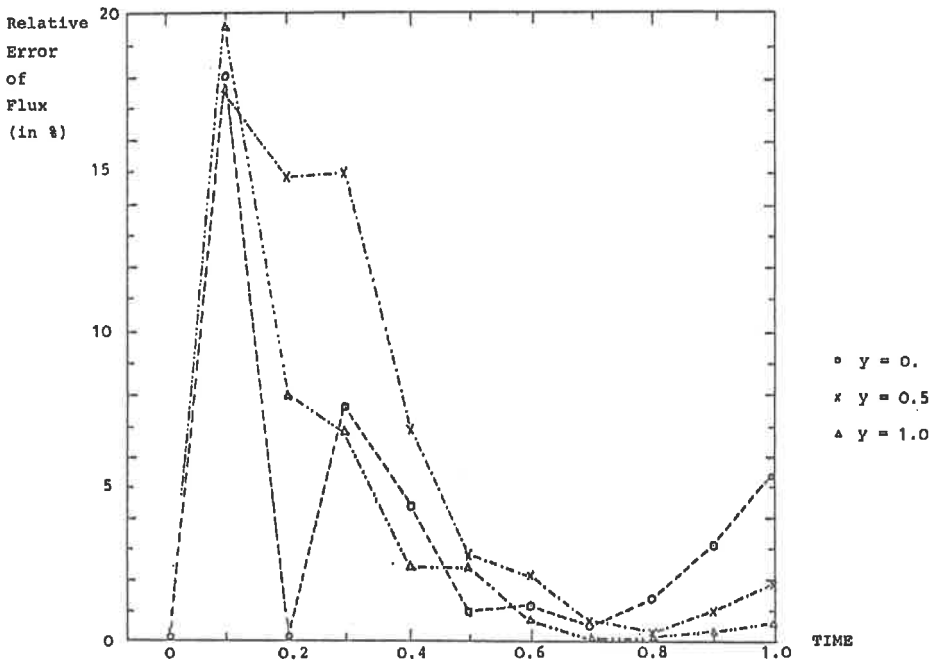


FIGURE 3 Computational error of the flux at x = 0
(Example as in Figure 2)

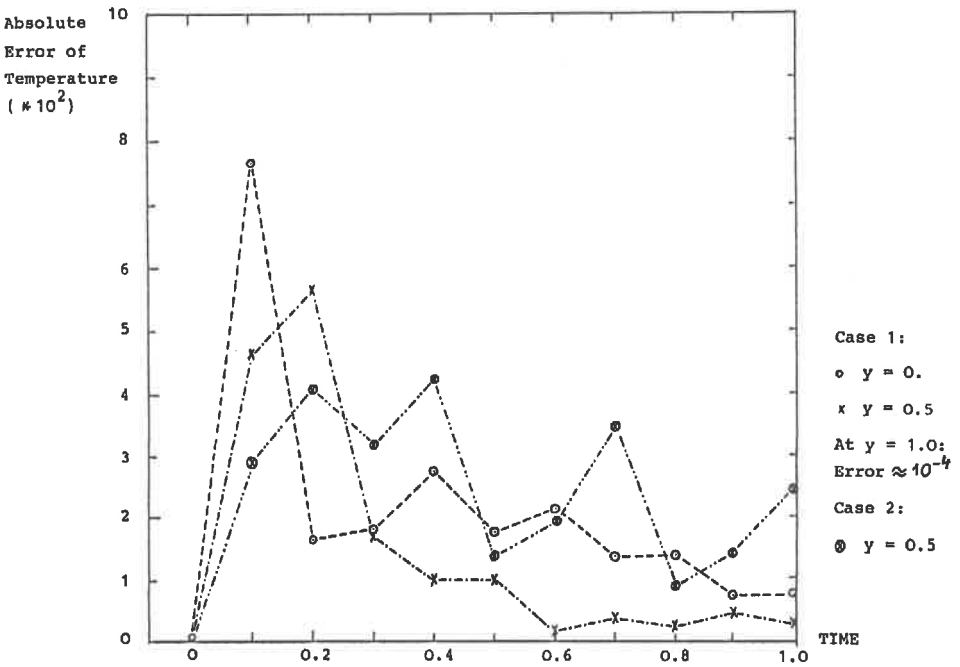


FIGURE 4 Computational error of temperature at x = 0; temperature data from truncated series (Case 1) and with perturbation of O(1.E-2) (Case 2)

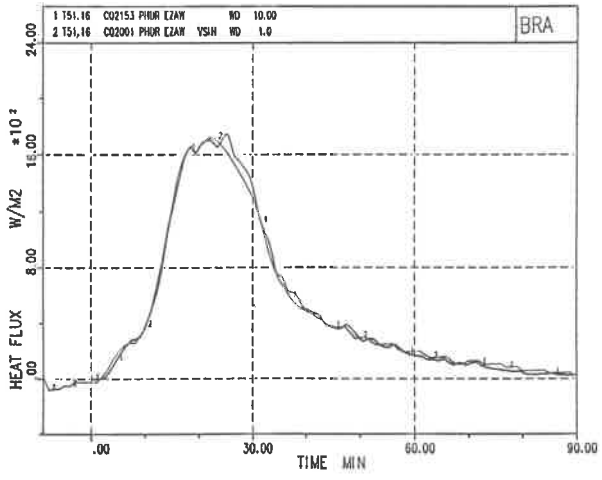


FIGURE 5

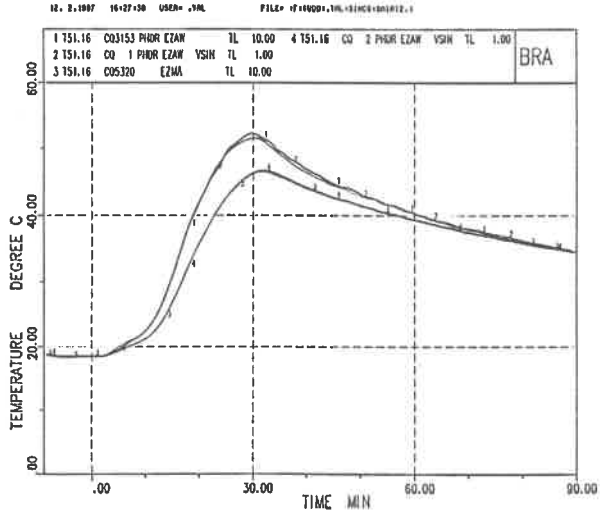


FIGURE 6

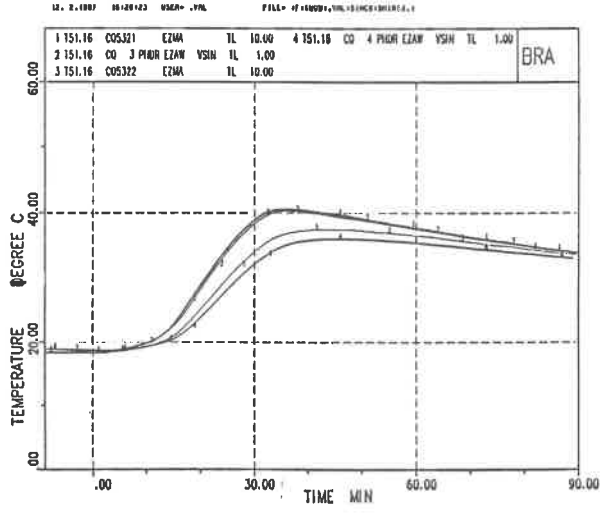


FIGURE 7