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A NEW LIMIT-LOAD FORMULA FOR LIGAMENT OF SURFACE CRACK IN LBB ASSESSMENT

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ABSTRACT

During Leak-Before-Break assessment, it is of great importance for predicting the limit-load of ligament of surface crack. The current limit-load formulas are almost over-conservative. By using them in LBB assessment, the actual transient length of through crack at penetration will be underestimated with unsafe results, which is not acceptable in engineering practice. In this paper, a new ligament collapse model of surface crack is proposed. The derivation of relevant new limit-load formula of ligament is briefly presented. Finally, authors' formula is compared with other formulas in detail via a lot of graphs.

1 INTRODUCTION

For the structure containing surface crack, especially for those thin-walled structure, the limit collapse of ligament is one of the essential rupture forms. Therefore, the study on limit-load formula of ligament is usually the important project for conventional fracture assessment, particularly for LBB assessment. The limit-load of ligament of surface crack is one of necessary criteria in assessing the behaviour of Leak-Before-Break by the transient-length estimation of through crack at penetration. Based on the experimental results and numerical calculations, many scholars proposed a lot of limit-load formulas with some assumptions. Nevertheless, all of those formulas are almost over-conservative. The ligament which may not break actually, will be predicted as collapse by use of those formulas. In the case of conventional fracture assessment, that results imply conservative evidently. But in the case of LBB assessment, the actual transient-length of through crack at penetration will be underestimated by those conservative limit-load formulas of ligament, and the detectable range of leakage will be extended correspondingly. As a result, an unsafe assessment would be presented that the actual Break-Before-Leak behaviour may be evaluated as Leak-Before-Break one. It is not acceptable in engineering practice.

In this paper, based on the experimental observation of fracture test with specimens containing surface crack, a new necking collapse model of ligament and the relevant derivation of new limit-load formula will be presented. Finally, authors' formula will be compared with other formulas in detail via a lot of graphs.

2 A NEW NECKING COLLAPSE MODEL OF LIGAMENT

For material of high toughness at the moment of ligament collapse the structure containing surface crack could be classified to three zones, i.e. the necking zone of ligament, the plastic zone surrounding necking and the elastic zone around the plastic area. The loading capacity of ligament depends on the size of necking zone and the plastic collapse characteristics of large deformation. Thus, authors' necking collapse model of ligament of surface crack on an infinite plate could be described as follows:

At the moment of necking collapse of ligament, the local necking zone is surrounded by wide elastic zone. In case, the normal displacement produced by remote load and cohesive stress on the interface of elastic zone and necking zone, is equal to the relative displacement of necking zone. It is called limit-rupture of the structure containing surface crack, and the corresponding remote load is called limit-load of that structure. Such model is shown in Fig 1.

Furthermore, five basic assumptions of model are stated below:

(1) The feature of surface crack at the moment of necking collapse is a iso-depth crack of length 2c and depth a;

(2) The necking zone of ligament is directly surrounded by wide elastic zone. In other words, the actual plastic zone between them is neglected in conservative consideration;

(3) The size of necking zone is $2c \times l_0 \times l_0$, where 2c is the length of surface crack; $l_0 = t - a$, the width of remaining ligament. The necking of ligament occurs on y-z plane only;

(4) Beyond the necking zone, the whole elastic plate is viewed as plane stress state, which is acceptable for thin-walled structure;

(5) The material in necking zone is incompressible, i.e. $\nu = 0.5$, $\varepsilon_m = 0$.

Based on previous assumptions, the derivation of limit-load formula could be simplified as the combination calculation of the elastic plate problem containing a rectangukar hole and the necking collapse problem of a rectangular strip as shown in Fig 1. Therefore, once the displacement δ_e at point A of interface is calculated by theoretical method, and the relative displacement δ_p at necking collapse is solved simultaneously, the limit-load formula could be derived by the condition of Eq(1):

$$\delta_e = \frac{1}{2}\delta_p \tag{1}$$

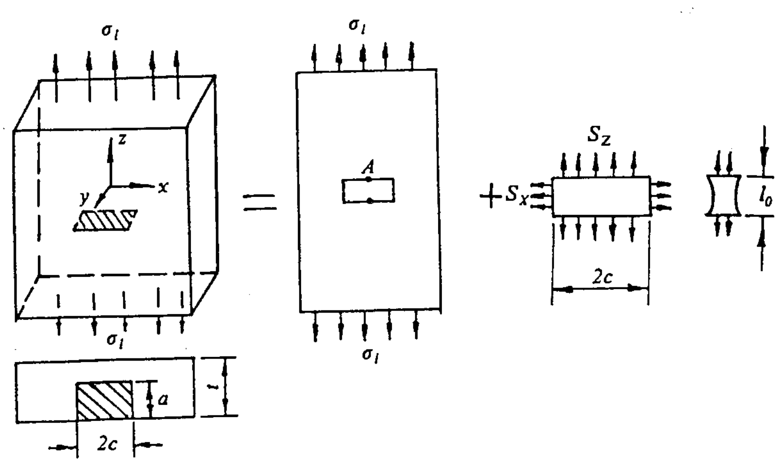
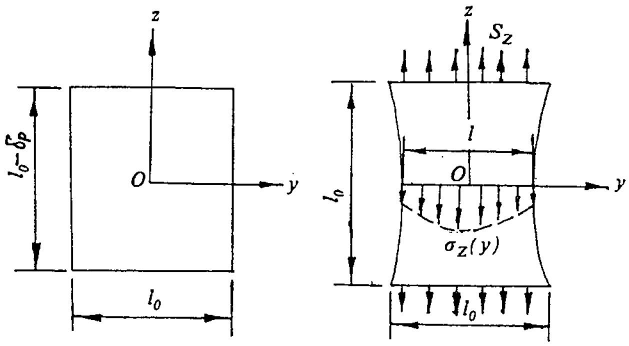


Fig 1 Necking collapse model of ligament

3 THE DERIVATION OF NEW LIMIT-LOAD FORMULA

3.1 The Necking Collapse Problem Of A Rectangular Strip

The necking problem of a rectangular strip is analysed referring to Bridgeman method of circular bar necking solution.[1] From previous assumption (3) the necking model is shown in Fig 2, where l_0 is the initial plate width, l is the transient plate width, δ_p is the relative displacement between upper and lower faces of necking zone, S_z is cohesive stress along z direction. The neck profile of free boundary will be approximately fitted with a hyperbola curve.



(a) (b)

Fig 2 Necking problem model of rectangular strip

At the root of necking (z = 0), the stress distribution along z direction could be written as:

$$\sigma_z = \frac{2}{\sqrt{3}}\sigma_e\left[1+\frac{1}{2}ln\left(1+\frac{l^2-4y^2}{2Rl}\right)\right] \tag{2}$$

where $\sigma_e=\frac{\sqrt{3}}{2}(\sigma_y-\sigma_z)$ is effective stress, which keeps constant along y direction ; R is the radius of curvature of hyperbola at z=0.
From the condition of static equilibrium along z direction in Fig 2(b), the cohesive stress S_z could be expressed as:

$$S_z = \frac{\sigma_e}{\sqrt{3}}\frac{l}{l_0}\left[\frac{2-\left(\frac{l}{l_0}\right)^2}{1-\left(\frac{l}{l_0}\right)^2}\right]ln\left\{3-2\left(\frac{l}{l_0}\right)^2+2\left[2-3\left(\frac{l}{l_0}\right)^2+\left(\frac{l}{l_0}\right)^4\right]^{1/2}\right\} \tag{3}$$

According to previous assumption (5), the area in Fig 2(a) is equal to that in Fig 2(b), the relative displacement is herefrom expressed as:

$$\delta_p = \frac{l_0}{2}\left\{1-\left(\frac{l}{l_0}\right)^2\left[1-\left(\frac{l}{l_0}\right)^2\right]^{-1/2}\left[-ln\left(\frac{l}{l_0}\right)+ln\left(1-\left(\frac{l}{l_0}\right)^2\right)^{1/2}\right]\right\} \tag{4}$$

Evidently, if l/l_0 at necking collapse is determined, both the cohesive stress and the relative displacement at necking collapse of ligament could easily be solved. Now, authors would recommend the l/l_0 formula as Eq(5) for plane strain state in accordance with the experimental results:

$$\frac{l}{l_0} = \sqrt{1-\psi} \tag{5}$$

where ψ is the shrinkage of section.

3.2 The Infinite Plate Problem Containing A Rectangular Hole Of Any Aspect Ratio

The model of elastic solution about the infinite plate problem containing a rectangular hole is shown in Fig 3. The distributing stress along the sides of rectangular hole is $q = l_0 S_z/t$, where t is the thickness of plate, S_z is cohesive stress determined from Eq(3). Provided that, a conformal map function of transforming rectangular hole into unit circle is found, the normal displacement δ_e at point A could easily be solved by complex function method of elasticity theory.

Now, an approximate map function has been recommended by authors in Ref [3]. In case $2c/l_0 = \lambda > 1$, the approximate conformal map function could be expressed as:

$$y = \omega(\zeta) = N\left[\frac{1}{\zeta} + cos2k\pi \cdot \zeta - \frac{1}{6}(1 - cos^2 2k\pi)\zeta^3\right] \tag{6}$$

where

$$N = c \,/ \left[\frac{5}{6} + cos2k\pi + \frac{1}{6}cos^2 2k\pi\right] \tag{7}$$

$$cos2k\pi = \frac{3 + 3\lambda - \sqrt{9(\lambda+1)^2 - 5(\lambda-1)^2}}{\lambda - 1} \tag{8}$$

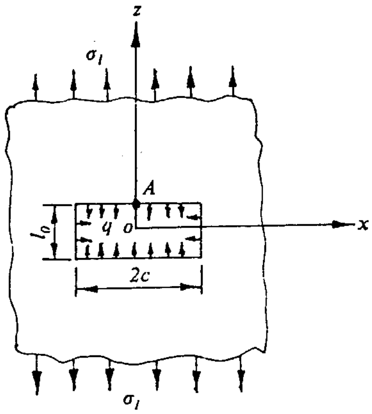


Fig 3 Elastic model of infinite plate problem containing rectangular hole

The elastic displacement at point A could thus be solved by complex function method as:

$$\begin{aligned}\delta_e = -\frac{1+\nu}{E}\Bigg\{&\frac{3-\nu}{1+\nu}\Bigg\{\left[-\frac{7}{24} + \frac{1}{4}cos2k\pi + \frac{1}{24}cos^2 2k\pi - \frac{3(1+cos2k\pi)}{7-cos^2 2k\pi}\right]\sigma_l \\ &+ \left[\frac{1}{6} - \frac{1}{6}cos^2 2k\pi + \frac{6cos2k\pi}{7-cos^2 2k\pi}\right]q\Bigg\} + \frac{5 - 6cos2k\pi + cos^2 2k\pi}{9 + 6cos2k\pi - 3cos^2 2k\pi}\Bigg\{\left[\frac{1}{8} + \frac{1}{4}cos2k\pi\right. \\ &\left. + \frac{1}{8}cos^2 2k\pi - \frac{3(1+cos2k\pi)}{7-cos^2 2k\pi}\right]\sigma_l + \left[\frac{1}{2} - \frac{1}{2}cos^2 2k\pi + \frac{6cos2k\pi}{7-cos^2 2k\pi}\right]q \\ &+ \left[-\left(\frac{5 + 4cos2k\pi - cos^2 2k\pi}{6 + 4cos2k\pi - 2cos^2 2k\pi}\right)\sigma_l + \left(\frac{2 + 2cos2k\pi}{3 + 2cos2k\pi - cos^2 2k\pi}\right)q\right]\Bigg\}N\end{aligned} \tag{9}$$

The detailed derivation of Eq(9) is presented in Ref [3].

3.3 Limit-Load Formula Of Ligament

By Eqs(2) and (9), authors' new limit-load formula of ligament of surface crack is expressed as:

$$\sigma_l = \left\{ \frac{1}{2}\frac{E}{1+\nu}\delta_p + \left[\frac{3-\nu}{1+\nu}\left(\frac{1}{6} - \frac{1}{6}cos^2 2k\pi + \frac{6cos2k\pi}{7-cos^2 2k\pi} \right) \right.\right.$$

$$+ \frac{2+2cos2k\pi}{3+2cos2k\pi - cos^2 2k\pi} + \frac{5-6cos2k\pi + cos^2 2k\pi}{9+6cos2k\pi - 3cos^2 2k\pi} \cdot$$

$$\left.\left.\left(\frac{1}{2} - \frac{1}{2}cos^2 2k\pi + \frac{6cos2k\pi}{7-cos^2 2k\pi} \right) \right] \frac{l_0}{t} S_z N \right\} \div \left\{ \frac{5+4cos2k\pi - cos^2 2k\pi}{6+4cos2k\pi - 2cos^2 2k\pi} \right.$$

$$+ \frac{3-\nu}{1+\nu}\left[\frac{7}{24} - \frac{1}{4}cos2k\pi - \frac{1}{24}cos^2 2k\pi + \frac{3(1+cos2k\pi)}{7-cos^2 2k\pi} \right]$$

$$\left. - \frac{5-6cos2k\pi + cos^2 2k\pi}{9+6cos2k\pi - 3cos^2 2k\pi}\left[\frac{1}{8} + \frac{1}{4}cos2k\pi + \frac{1}{8}cos^2 2k\pi - \frac{3(1+cos2k\pi)}{7-cos^2 2k\pi} \right] \right\} N \qquad (10)$$

where S_z , δ_p , N and cos2kπ are determined from Eqs(3), (4), (7) and (8) respectively; ν is Poisson's ratio.

4 COMPARISON AND CONCLUSION

The current limit-load formulas reported in literatures are Harrison[4], Chell [5], Mattheck[6], Harrop[7] and Miller[8] etc. Among them the Miller's formula was set up on the basis of experimental data within a range of material constant k = 0.3~1.3.

A comparison between authors' formula and current formulas via a lot of graph is completed with steel 16MnR. From Fig 4~7 we may conclude that:

(1) The results of authors' formula is very near to that of lower bound of Miller's formula (k=0.3). When a/c=0.4 both curves are almost coincident. However, in the case of a/c < 0.4 authors' results tend to be greater than Miller's lower bound, but less than his upper bound. It implies that the accuracy of authors' formula is better than that of Miller's.

(2) In any case, the formulas of Harrison, Mattheck, Chell and Harrop are rather conservative; especially in the case of a/t < 0.9, it is much more conservative than authors'.

(3) When remote stress is less than 0.5σ_y, the local ligament collapse will occur at deeper and longer surface crack only. This conclusion is agree with Smith's report in Ref [9].

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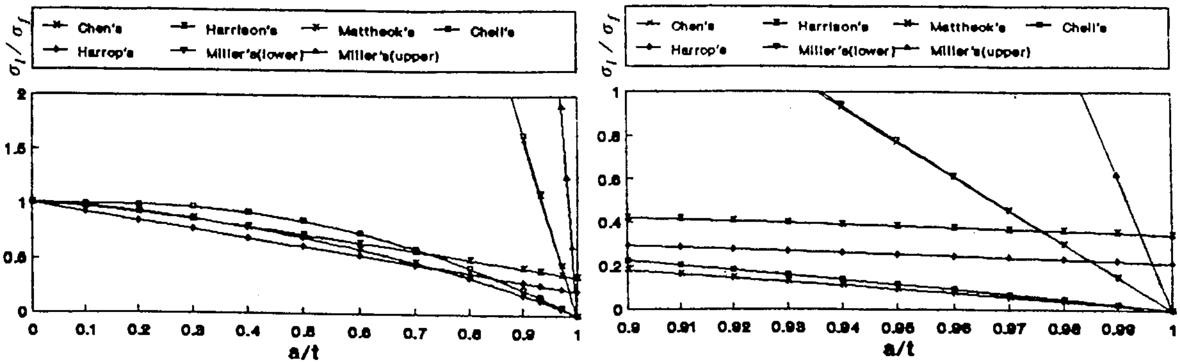

Fig 4 Comparison of ligament limit-load (a/c = 0.4)

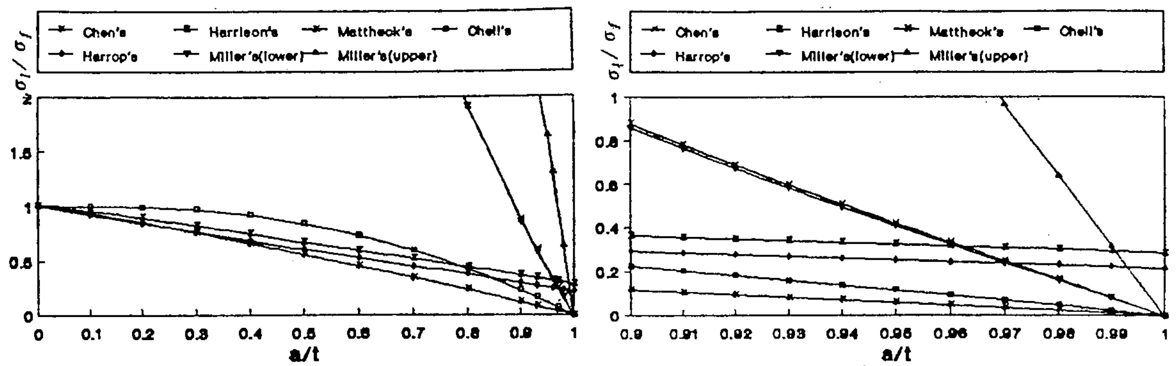

Fig 5 Comparison of ligament limit-load (a/c = 0.2)

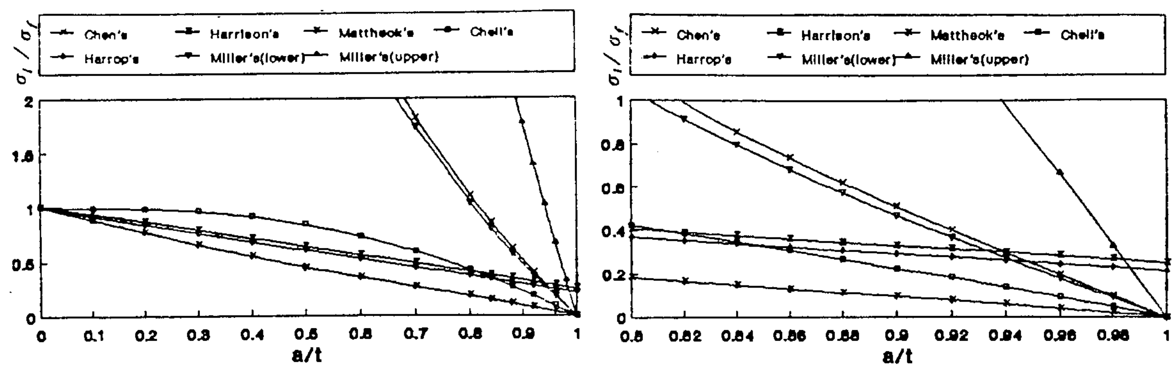

Fig 6 Comparison of ligament limit-load (a/c = 0.1)

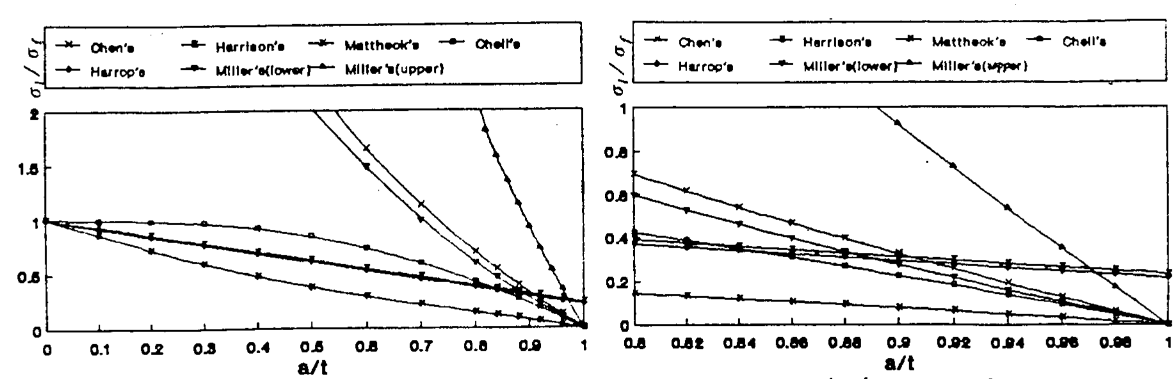

Fig 7 Comparison of ligament limit-load (a/c = 0.05)

Note: σ_l - limit-load of ligament; σ_f - flow stress of material