Solution of Boundary and Mixed Problems in Arbitrary Domains Using Fourier Transforms

Gilnei Gonçalves Furtado[1)], Marco Túllio de Vilhena[2)], Telmo Roberto Strohaecker[3)] and Rubem Panta Pazos[4)]

1) PPGEM/UFRGS, R. Oswaldo Aranha, 99, 6° andar, Porto Alegre / RS, 90035-190, Brazil, gilnei@demet.ufrgs.br
2) PPGEM/UFRGS, E-mail: vilhena@mat.ufrgs.br
3) PPGEM/UFRGS, E-mail: telmo@demet.ufrgs.br
4) FAMAT/PUCRS, E-mail: rpp@pucrs.br

ABSTRACT

In this work, we develop a method to solve partial differential equations in arbitrary domains by applying Fourier Transform technique. The main target of this work are the linear elastic problem. For such, the Fourier Transform technique is developed in domains with compact support, i.e. with the assumption that the unknown vanishes outside the domain. This approach establish an alternative procedure to determine the integral formulation for the boundary element method. We report solutions for the two-dimensional elastic linear equation.

INTRODUCTION

In a recent work Furtado, Vilhena and Strohaecker [1] proposed a closed-form solution for the general linear partial differential equation with constant coefficients defined over a finite domain using the Fourier transform technique. Briefly speaking, the Fourier transform was applied to this problem, assuming that the solution vanishes outside the domain. After solving the transformed equation, the closed-form solution was obtained making the Fourier transform inversion.

In this work, we apply this approach to solve the two-dimensional linear elastic problem considering small strains, planar stress and isotropic material. To reach this goal, we outline the paper as follows: initially, we present the closed-form solution for the elastic problem. Afterward we report comments and simulation for the stress.

TWO-DIMENSIONAL LINEAR ELASTIC PROBLEM

We express the two-dimensional linear elastic problem considering small strains, planar stress and isotropic material, by the following partial differential equations

$$B\frac{\partial^2 u}{\partial x^2} + (B-G)\frac{\partial^2 v}{\partial x\,\partial y} + G\frac{\partial^2 u}{\partial y^2} = 0\,, \tag{1}$$

$$G\frac{\partial^2 v}{\partial x^2} + (B-G)\frac{\partial^2 u}{\partial x\,\partial y} + B\frac{\partial^2 v}{\partial y^2} = 0\,, \tag{2}$$

defined over an arbitrary finite domain $\Omega \subset \Re^2$ (figure (1)), where u and v are displacements in the x and y directions, and B and G are constants that depend on elastic material properties ($G = \frac{E}{2(1+\nu)}$ and $B = \frac{E}{1-\nu^2}$, E is the modulus of elasticity and ν is Poisson's ratio) [2].

To solve this problem applying the Fourier Transform, we follow the idea of Furtado et al [1]. To this end, we begin with the application of the Fourier transform technique defined over finite domain in Eq. (1) and (2). It turns out:

$$\begin{aligned}\overline{u} \;=\; & -\frac{1}{2\pi}\frac{1}{BG(r^2+s^2)^2}\Bigg\{-(Gr^2+Bs^2)\Bigg[B\oint_\Gamma \frac{\partial u}{\partial x}e^{i(rx+sy)}dy - G\oint_\Gamma \frac{\partial u}{\partial y}e^{i(rx+sy)}dx \\ & -irB\oint_\Gamma u\,e^{i(rx+sy)}dy + isG\oint_\Gamma u\,e^{i(rx+sy)}dx + (B-G)\oint_\Gamma \frac{\partial v}{\partial y}e^{i(rx+sy)}dy \\ & +ir(B-G)\oint_\Gamma v\,e^{i(rx+sy)}dx\Bigg] + (B-G)rs\Bigg[(B-G)\oint_\Gamma \frac{\partial u}{\partial y}e^{i(rx+sy)}dy \\ & +ir(B-G)\oint_\Gamma u\,e^{i(rx+sy)}dx - B\oint_\Gamma \frac{\partial v}{\partial y}e^{i(rx+sy)}dx + G\oint_\Gamma \frac{\partial v}{\partial x}e^{i(rx+sy)}dy \\ & +isB\oint_\Gamma v\,e^{i(rx+sy)}dx - irG\oint_\Gamma v\,e^{i(rx+sy)}dy\Bigg]\Bigg\}\end{aligned} \tag{3}$$

and

$$\begin{aligned}
\overline{v} \;=\; & -\frac{1}{2\pi}\frac{1}{BG(r^2+s^2)^2}\left\{(B-G)rs\left[B\oint_\Gamma \frac{\partial u}{\partial x}e^{i(rx+sy)}dy - G\oint_\Gamma \frac{\partial u}{\partial y}e^{i(rx+sy)}dx\right.\right.\\
& -irB\oint_\Gamma u\,e^{i(rx+sy)}dy + isG\oint_\Gamma u\,e^{i(rx+sy)}dx + (B-G)\oint_\Gamma \frac{\partial v}{\partial y}e^{i(rx+sy)}dy\\
& \left.+ir(B-G)\oint_\Gamma v\,e^{i(rx+sy)}dx\right] - (Br^2+Gs^2)\left[(B-G)\oint_\Gamma \frac{\partial u}{\partial y}e^{i(rx+sy)}dy\right.\\
& +ir(B-G)\oint_\Gamma u\,e^{i(rx+sy)}dx - B\oint_\Gamma \frac{\partial v}{\partial y}e^{i(rx+sy)}dx + G\oint_\Gamma \frac{\partial v}{\partial x}e^{i(rx+sy)}dy\\
& \left.\left.+isB\oint_\Gamma v\,e^{i(rx+sy)}dx - irG\oint_\Gamma v\,e^{i(rx+sy)}dy\right]\right\} \qquad (4)
\end{aligned}$$

For sake of simplification, we define the ensuing Fourier transformed functions

$$\overline{F_1}(r,s) = \frac{Gr^2+Bs^2}{BG\left(r^2+s^2\right)^2} \qquad (5)$$

$$\overline{F_2}(r,s) = \frac{Br^2+Gs^2}{BG\left(r^2+s^2\right)^2} \qquad (6)$$

$$\overline{H}(r,s) = \frac{(B-G)rs}{BG\left(r^2+s^2\right)^2} \qquad (7)$$

Here we adopt the notation: $\overline{F_1}$, $\overline{F_2}$ and $\overline{H}$ stand for the Fourier transform of the functions F_1, F_2 and H. Furthermore, $\overline{F_{1_x}}$, $\overline{F_{1_y}}$, $\overline{F_{2_x}}$, $\overline{F_{2_y}}$, $\overline{H_x}$ and $\overline{H_y}$ denote partial derivatives respect to variables x and y.

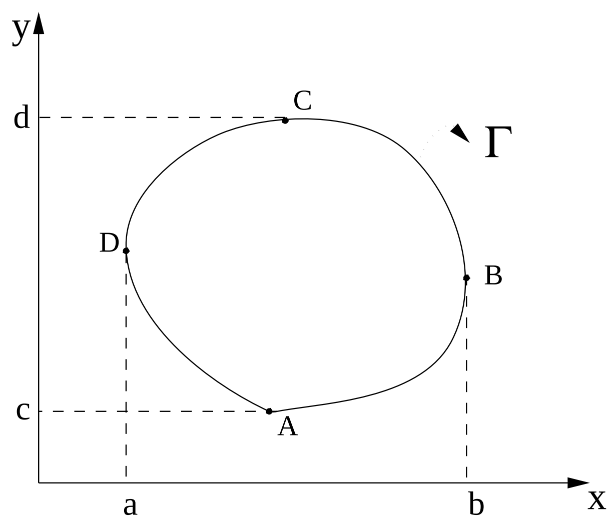


Figure 1: Domain Ω.

Next we perform the inverse Fourier transform of the functions $\overline{F_{1_x}}$, $\overline{F_{1_y}}$, $\overline{F_{2_x}}$, $\overline{F_{2_y}}$, $\overline{H_x}$ and $\overline{H_y}$. For such, applying the theorem of residue we get:

$$F_{1_x}(x,y) = \frac{1}{2GB}((B-3G)y^2-(G+B)x^2)\frac{x}{(x^2+y^2)^2} \qquad (8)$$

$$F_{1_y}(x,y) = \frac{1}{2GB}((G-3B)x^2-(B+G)y^2)\frac{y}{(x^2+y^2)^2} \qquad (9)$$

$$F_{2_x}(x,y) = \frac{1}{2GB}((G-3B)y^2-(B+G)x^2)\frac{x}{(x^2+y^2)^2} \qquad (10)$$

$$
\begin{aligned}
F_{2y}(x,y) &= \frac{1}{2GB}((B-3G)x^2-(G+B)y^2)\frac{y}{(x^2+y^2)^2} && (11)\\
H_x(x,y) &= \frac{B-G}{2BG}\;\frac{x^2y-y^3}{(x^2+y^2)^2} && (12)\\
H_y(x,y) &= \frac{B-G}{2BG}\;\frac{-x^3+x\,y^2}{(x^2+y^2)^2} && (13)
\end{aligned}
$$

On the other hand, integrating Eq. (8)—(13), we readily obtain the functions F_1, F_2 and H:

$$
\begin{aligned}
F_1(x,y) &= \frac{1}{2GB}\left[-\frac{G+B}{2}\mathbf{ln}(x^2+y^2)+\frac{B-G}{2}\frac{x^2-y^2}{x^2+y^2}\right] && (14)\\
F_2(x,y) &= \frac{1}{2GB}\left[-\frac{G+B}{2}\mathbf{ln}(x^2+y^2)+\frac{B-G}{2}\frac{y^2-x^2}{x^2+y^2}\right] && (15)\\
H(x,y) &= -\frac{B-G}{2BG}\;\frac{xy}{x^2+y^2} && (16)
\end{aligned}
$$

Now, we are in position to write down a closed-form solution for the problem (1)—(2). We reach this goal, making the Fourier inversion of $\overline{u}$ and $\overline{v}$ given by Eq. (3) and (4). This procedure leads to

$$
\begin{aligned}
u^*(x_a,y_a) = -\frac{1}{2\pi}\Bigg\{ & -B\oint_\Gamma \frac{\partial u}{\partial x}F_1(x_a-x,y_a-y)dy + G\oint_\Gamma \frac{\partial u}{\partial y}F_1(x_a-x,y_a-y)dx \\
& -B\oint_\Gamma u\,F_{1_x}(x_a-x,y_a-y)dy + G\oint_\Gamma u\,F_{1_y}(x_a-x,y_a-y)dx \\
& -(B-G)\oint_\Gamma \frac{\partial v}{\partial y}F_1(x_a-x,y_a-y)dy + (B-G)\oint_\Gamma v\,F_{1_x}(x_a-x,y_a-y)dx \\
& +(B-G)\oint_\Gamma \frac{\partial u}{\partial y}H(x_a-x,y_a-y)dy - (B-G)\oint_\Gamma u\,H_x(x_a-x,y_a-y)dx \\
& -B\oint_\Gamma \frac{\partial v}{\partial y}H(x_a-x,y_a-y)dx + G\oint_\Gamma \frac{\partial v}{\partial x}H(x_a-x,y_a-y)dy \\
& -B\oint_\Gamma v\,H_y(x_a-x,y_a-y)dx + G\oint_\Gamma v\,H_x(x_a-x,y_a-y)dy\Bigg\} \qquad (17)
\end{aligned}
$$

and

$$
\begin{aligned}
v^*(x_a,y_a) = -\frac{1}{2\pi}\Bigg\{ & B\oint_\Gamma \frac{\partial u}{\partial x}H(x_a-x,y_a-y)dy - G\oint_\Gamma \frac{\partial u}{\partial y}H(x_a-x,y_a-y)dx \\
& +B\oint_\Gamma u\,H_x(x_a-x,y_a-y)dy - G\oint_\Gamma u\,H_y(x_a-x,y_a-y)dx \\
& +(B-G)\oint_\Gamma \frac{\partial v}{\partial y}H(x_a-x,y_a-y)dy - (B-G)\oint_\Gamma v\,H_x(x_a-x,y_a-y)dx \\
& -(B-G)\oint_\Gamma \frac{\partial u}{\partial y}F_2(x_a-x,y_a-y)dy + (B-G)\oint_\Gamma u\,F_{2_x}(x_a-x,y_a-y)dx \\
& +B\oint_\Gamma \frac{\partial v}{\partial y}F_2(x_a-x,y_a-y)dx - G\oint_\Gamma \frac{\partial v}{\partial x}F_2(x_a-x,y_a-y)dy \\
& +B\oint_\Gamma v\,F_{2_y}(x_a-x,y_a-y)dx - G\oint_\Gamma v\,F_{2_x}(x_a-x,y_a-y)dy\Bigg\} \qquad (18)
\end{aligned}
$$

Recasting the above ansatz in matrix fashion we have

$$
\mathbf{U}^* = \oint_\Gamma \mathbf{A}\,\mathbf{U}\,d\tau + \oint_\Gamma \mathbf{B}\,\partial\mathbf{U}\,d\tau \qquad (19)
$$

where $\mathbf{U}^*$, $\mathbf{U}$ and $\partial\mathbf{U}$ are the vectors

$$\mathbf{U}^* = \mathbf{col}\,(u^*(x_a, y_a),\ v^*(x_a, y_a)) \tag{20}$$

$$\mathbf{U} = \mathbf{col}\,(u(x, y),\ v(x, y)) \tag{21}$$

$$\partial\mathbf{U} = \mathbf{col}\left(\frac{\partial}{\partial x}u(x,y),\ \frac{\partial}{\partial y}u(x,y),\ \frac{\partial}{\partial x}v(x,y),\ \frac{\partial}{\partial y}v(x,y)\right) \tag{22}$$

and the vectors $\mathbf{A}$ and $\mathbf{B}$ have the entries

$$\mathbf{A} = -\frac{1}{2\pi}\begin{pmatrix} -B\,F_{1_x}c_x - G\,F_{1_y}c_y + (B-G)H_x c_y & -(B-G)F_{1_x}c_y + B\,H_y c_y + GH_x c_x \\ +B\,H_x c_x + G\,H_y c_y - (B-G)F_{2_x}c_y & +(B-G)H_x c_y - B\,F_{2_y}c_y - G\,F_{2_x}c_x \end{pmatrix} \tag{23}$$

$$\mathbf{B} = -\frac{1}{2\pi}\begin{pmatrix} -B\,F_1 c_x & -G\,F_1 c_y + (B-G)H\,c_x & +G\,H\,c_x & -(B-G)F_1 c_x + B\,H\,c_y \\ +B\,H\,c_x & +G\,H\,c_y - (B-G)F_2 c_x & -G\,F_2\,c_x & +(B-G)H\,c_x - B\,F_2 c_y \end{pmatrix} \tag{24}$$

evaluated at $x_a - x$ and $y_a - y$. Here $d\tau$ is given by

$$\begin{aligned} -c_y\,d\tau &= dx \\ +c_x\,d\tau &= dy \end{aligned} \tag{25}$$

and $c_x,\ c_y$ are the director cosines of the normal to the contour of the domain.

Noticing that the boundary conditions are prescribed for the force intensity at boundary, we consider the ensuing relationship between $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial \tau}$, $\frac{\partial v}{\partial \tau}$, f_x, f_y where f_x, f_y are the force intensities at boundary in the x and y directions and $\frac{\partial u}{\partial \tau}$, $\frac{\partial v}{\partial \tau}$ are tangential derivatives of u and v. Writing this relationship in matrix notation we have

$$\mathbf{W} = \mathbf{C}\,\partial\mathbf{U} \tag{26}$$

where the matrix $\mathbf{C}$ has the entries

$$\mathbf{C} = \begin{pmatrix} -c_y & +c_x & 0 & 0 \\ 0 & 0 & -c_y & +c_x \\ +B\,c_x & +G\,c_y & +G\,c_y & +(B-2G)\,c_x \\ +(B-2G)c_y & +G\,c_x & +G\,c_x & +B\,c_y \end{pmatrix} \tag{27}$$

and the column vectors $\mathbf{U}_\tau$, $\mathbf{F}$ and $\mathbf{W}$ are given by

$$\mathbf{U}_\tau = \mathbf{col}\left(\frac{\partial u}{\partial \tau}, \frac{\partial v}{\partial \tau}\right) \tag{28}$$

$$\mathbf{F} = \mathbf{col}\,(f_x,\ f_y) \tag{29}$$

$$\mathbf{W} = \mathbf{col}\,(\mathbf{U}_\tau,\ \mathbf{F}) \tag{30}$$

Replacing Eq. (26) in Eq. (19) we obtain

$$\mathbf{U}^* = \oint_\Gamma \mathbf{A}\,\mathbf{U}\,d\tau + \oint_\Gamma \mathbf{B}\,\mathbf{C}^{-1}\,\mathbf{W}\,d\tau \tag{31}$$

We define

$$\mathbf{D} = \mathbf{B}\,\mathbf{C}^{-1} \tag{32}$$

and rewrite $\mathbf{D}$ as block matrix

$$\mathbf{D} = (\mathbf{D}_1,\ \mathbf{D}_2) \tag{33}$$

where $\mathbf{D}$ is 2×4 matrix and $\mathbf{D}_1$ and $\mathbf{D}_2$ are 2×2 matrices. Therefore, the Eq. (31), using this definition, has the form

$$\mathbf{U}^* = \oint_\Gamma [\mathbf{A}\,\mathbf{U} + \mathbf{D}_1\,\mathbf{U}_\tau]\,d\tau + \oint_\Gamma \mathbf{D}_2\,\mathbf{F}\,d\tau \tag{34}$$

which is an appropriate closed-form solution when displacement and force are prescribed at boundary.

Next, we simplify the Eq. (34). For such using the result

$$\oint_{\Gamma} \mathbf{D}_1\, \mathbf{U}_{\tau}\, d\tau = -\oint_{\Gamma} \frac{\partial}{\partial \tau} \mathbf{D}_1\, \mathbf{U}\, d\tau \tag{35}$$

combined with the definition

$$\mathbf{E} = \mathbf{A} - \frac{\partial}{\partial \tau} \mathbf{D}_1 \tag{36}$$

we quote the Eq. (34) as

$$\mathbf{U}^{*} = \oint_{\Gamma} \mathbf{E}\, \mathbf{U}\, d\tau + \oint_{\Gamma} \mathbf{D}_2\, \mathbf{F}\, d\tau \tag{37}$$

To this point it is important to remember that u^{*} and v^{*} coincide with u and v inside the domain Ω and vanish outside the domain Ω.

In order to get the unknown boundary conditions for the u and v components of $\mathbf{U}$ and for the f_x and f_y components of $\mathbf{F}$ we proceed in similar manner as in Boundary Integral Method. In what follows, we briefly discuss this procedure:

1. We firstly begin expanding the components of $\mathbf{U}$ and $\mathbf{F}$ in terms of some known basis functions;
2. We select a set of N auxiliary points (positioned inside or outside the domain), where N is the number of unknown coefficients in the expansion of $\mathbf{U}$ and $\mathbf{F}$ components. For more details see Furtado et all [1];
3. Finally, we set up the linear system to determine the expansion coefficients, substituting the functions basis expansion into Eq. (37) (or (34)) and evaluating this equation at N auxiliary points.

NUMERICAL SIMULATION AND CONCLUSION

To illustrate the capability of the proposed approach to solve two-dimensional linear elastic problem, let us consider the problem and boundary conditions defined in the domain, depicted in figure (2). The material properties are $E = 210\, GPa$, for elasticity modulus and $\nu = 0.3$, for Poisson's coefficient. Furthermore, we assume as basis functions, first degree polynomials and evaluate the unknown functions u, v, f_x and f_y at boundary using Eq. (37). To this point, it is important to mention that the auxiliary points are positioned outside the domain, apart from the nodes at boundary, a distance of 20% the length of the discretization step. This procedure allow us to avoid the singularity in the integral evaluated at boundary. In order to show the numerical convergence of the proposed approach we report numerical results for the following averaged distances between nodes 10, 5 and 2.5 mm and we compare the results with the ones attained by the finite element method (ABAQUS version 5.8.1). For the finite element results we take advantage of the problem symmetry. However, symmetry can not be considered in our approach because the nodes at the corner of the domain. Numerical comparisons for the maximum and minimum values for u and v ($x \geq 0$) as well the values of the stress σ_{xx}, σ_{yy} and σ_{xy}, the values of stress σ_{xx} at points A and B (see figure (2)) and the values of displacements u and v at point C (see figure (2)) are reported in tables (1) to (3). From the previous results we readily notice the numerical convergence of the results encountered by this method and the good comparison with the ones achieved by the finite elements method, when we refine the mesh size of the discretized domain. For sake of completeness in figure (3) we report the results for stress distribution in x direction by discussed methods for the mesh size of $2.5mm$. Given a closer look in these results we promptly observe a very good agreement.

Now, let us assume assume that the considered problem has unique solution. Furthermore, we have previously shown that the achieved integral formulation for the solution, through the Fourier Transform technique, is the solution of the problem under consideration. Therefore, we are confident to affirm that the results encountered, actually are solutions of the problem discussed. This argument is reinforced by the good coincidence with finite elements results.

Concluding, we would like to point out that we do believe that the Fourier Transform technique is a promising and alternative approach to generate the integral formulation of Boundary Integral method. Indeed in our opinion this method is quite simple and straight as well general in the sense that it can be applied to solve partial differential equations with even and odd order. It is also important to remark that all the numerical values for displacement and stress are very close to zero outside the domain satisfying in this manner the hypothesis of the zero value for the solution out side the domain (see figure (3)). Recalling that we have already proved the equivalence between the Fourier transform and variational methods to obtain the integral closed-form solution for the heat conduction problem [1][3], we focus our future work to the task of also showing the equivalence for the discussed problem as well to the subject of proving the convergence, under mathematical point of view, of the proposed methods. Application of this approach to solve partial differential equation with odd order is also under investigation.

Table 1: Maximum and minimum for the displacement u and v and for the stress σ_{xx}, σ_{yy} and σ_{xy}.

Discretization step (mm)	Displacement $u\,(mm)$			
	Maximum		Minimum	
	BEM	FEM	BEM	FEM
10.0	$+8.9315e-1$	$+8.64e-1$	$-4.8537e-2$	$-4.69e-2$
5.0	$+8.9502e-1$	$+8.87e-1$	$-4.8327e-2$	$-4.79e-2$
2.5	$+8.9450e-1$	$+8.94e-1$	$-4.8341e-2$	$-4.83e-2$
Discretization step	Displacement $v\,(mm)$			
	Maximum		Minimum	
	BEM	FEM	BEM	FEM
$10.0mm$	$+8.3219e-2$	$+8.14e-2$	$-3.4855e-1$	$-3.37e-1$
$5.0mm$	$+8.3467e-2$	$+8.32e-2$	$-3.4889e-1$	$-3.46e-1$
$2.5mm$	$+8.3390e-2$	$+8.35e-2$	$-3.4860e-1$	$-3.48e-1$
Discretization step	Stress $\sigma_{xx}\,(MPa)$			
	Maximum		Minimum	
	BEM	FEM	BEM	FEM
$10.0mm$	$+8.7720e+2$	$+8.63e+2$	$-2.7212e+2$	$-2.65e+2$
$5.0mm$	$+8.5678e+2$	$+8.63e+2$	$-2.7549e+2$	$-2.73e+2$
$2.5mm$	$+8.4592e+2$	$+8.54e+2$	$-2.7657e+2$	$-2.76e+2$
Discretization step	Stress $\sigma_{yy}\,(MPa)$			
	Maximum		Minimum	
	BEM	FEM	BEM	FEM
$10.0mm$	$+4.8501e+2$	$+4.69e+2$	$-2.1027e+2$	$-1.98e+2$
$5.0mm$	$+4.8456e+2$	$+4.59e+2$	$-2.0908e+2$	$-2.06e+2$
$2.5mm$	$+4.8421e+2$	$+4.80e+2$	$-2.0841e+2$	$-2.09e+2$
Discretization step	Stress $\sigma_{xy}\,(MPa)$			
	Maximum		Minimum	
	BEM	FEM	BEM	FEM
$10.0mm$	$+3.5722e+2$	$+2.17e+2$	$-1.2273e+2$	$-1.21e+2$
$5.0mm$	$+3.6345e+2$	$+3.14e+2$	$-1.2393e+2$	$-1.24e+2$
$2.5mm$	$+3.6303e+2$	$+3.36e+2$	$-1.2410e+2$	$-1.24e+2$

Table 2: Stress σ_{xx} at positions $(0,-25)$ and $(0,-100)$.

Location		Discretization step	$\sigma_{xx}\,(MPa)$	
x	y		BEM	FEM
$0.0mm$	$-25.0mm$	$10.0mm$	$+8.7720e+2$	$+8.627e+2$
		$5.0mm$	$+8.5678e+2$	$+8.625e+2$
		$2.5mm$	$+8.4592e+2$	$+8.539e+2$
$0.0mm$	$-100.0mm$	$10.0mm$	$-2.7021e+2$	$-2.646e+2$
		$5.0mm$	$-2.7377e+2$	$-2.727e+2$
		$2.5mm$	$-2.7521e+2$	$-2.760e+2$

Table 3: Displacements u and v at positions $(62.5, 120)$.

Location		Discretization step	$u\,(mm)$		$v\,(mm)$	
x	y		BEM	FEM	BEM	FEM
$62.5mm$	$120.0mm$	$10.0mm$	$+8.9281e-1$	$+8.640e-1$	$-1.3459e-1$	$-1.302e-1$
		$5.0mm$	$+8.9465e-1$	$+8.864e-1$	$-1.3469e-1$	$-1.618e-1$
		$2.5mm$	$+8.9411e-1$	$+8.932e-1$	$-1.3459e-1$	$-1.344e-1$

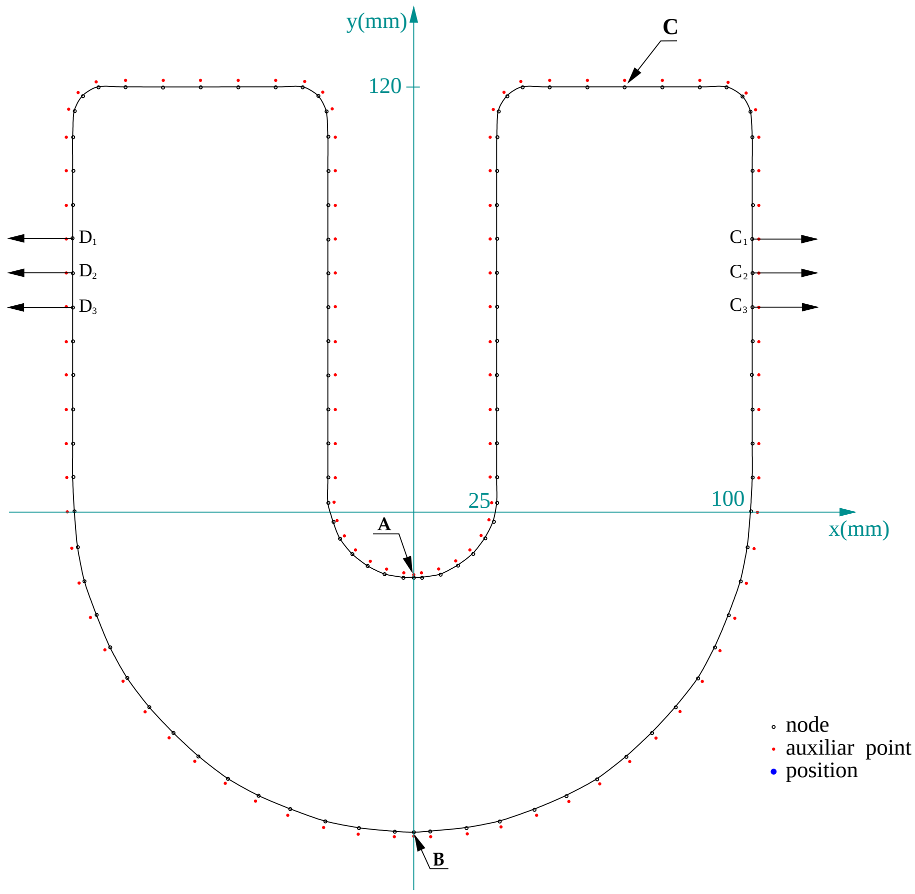
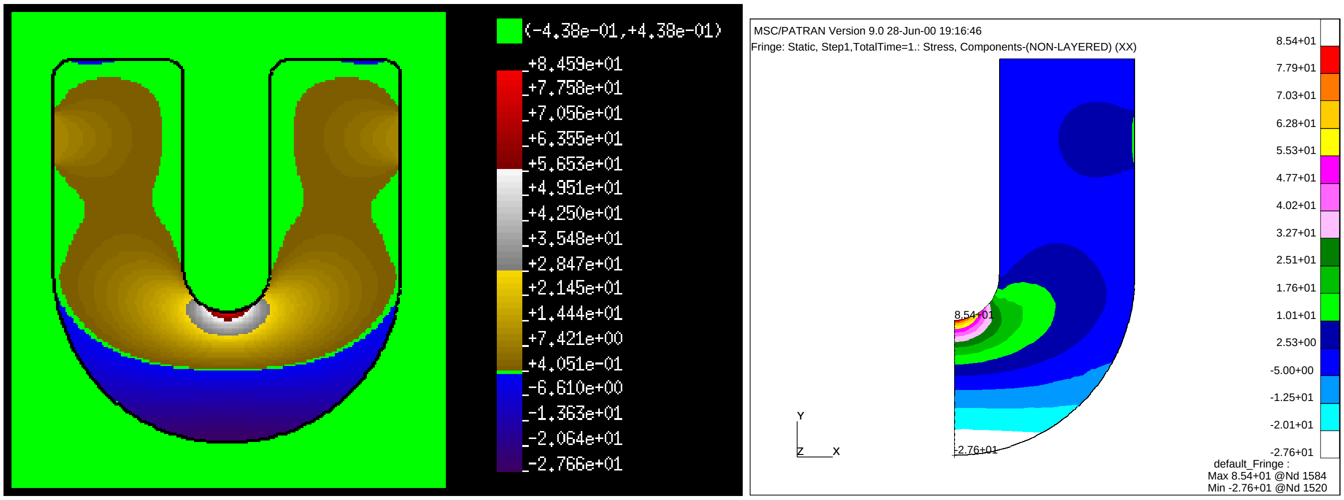


Figure 2: Discretized domain.

Boundary conditions at nodes: $\mathbf{A}(u = 0, f_y = 0)$, $\mathbf{B}(u = 0, v = 0)$, $\mathbf{C}_{1,2,3}(f_x = +100\,MPa,\ f_y = 0)$, $\mathbf{D}_{1,2,3}(f_x = -100\,MPa,\ f_y = 0)$ and the remaining nodes $(f_x = 0, f_y = 0)$.



(a) BEM. (b) FEM.

Figure 3: Color map for stress distribution σ_{xx} (color scales $\times 10\ MPa$) obtained for $2.5mm$ size step.

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