ABSTRACT

WHITE, TRACY FOOTE. Teacher Questioning in Undergraduate Mathematics: A Collective Case Study. (Under the direction of Dr. Allison McCulloch).

This study examines the mathematical questioning of undergraduate Calculus I instructors for the purpose of detailing the ways in which instructors are using their questions. The emphasis is on verbal questions because of their in-the-moment value and ability to get students engaged in discourse. Calculus I is of particular interest because of its impact of the retention of Science, Technology, Engineering, and Mathematics (STEM) majors. I investigated the types of questions five instructors asked, the ways they used wait time, and the classroom setting during instruction. I found that the categories used to describe the types of questions posed in secondary mathematics classes (Boaler & Brodie, 2004) were appropriate for detailing the types of questions undergraduate instructors ask. Moreover, I found that the instructors asked a variety of question types in at least two in-class activities and most were just as likely to use wait time as they were to use no wait time. While there were similarities in the instructors' overall questioning, the cases stand alone as unique examples of the ways undergraduate Calculus I use their questions.

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Teacher Questioning in Undergraduate Mathematics: A Collective Case Study

by
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DEDICATION

I dedicate this dissertation to my father, Rickie Foote, who has watched me from above providing motivation and strength throughout the entire PhD process. It is also dedicated to my mother, Linda Perkins Foote, and my husband Nathan White, Jr. who have been my number one cheerleaders, supporters, and voices of reason. Lastly, this dissertation is dedicated to my family and friends with special attention devoted to my cousin, Stacy Ingram Andrews, and my father-n-law, Nathan White, Sr. who passed months before I walked across the stage to get my degree.

BIOGRAPHY

Tracy Foote White was born on January 27, 1978 to Linda Perkins Foote and the late Rickie W. Foote. She is a native of Winston Salem, North Carolina, the very place she completed all of her education prior to her terminal degree. In 1996, she graduated from North Forsyth High School. Tracy then matriculated at Winston Salem State University and graduated summa cum laude with a Bachelor of Science in Mathematics with Secondary Teaching Certification in May 2000. Upon graduation, she began teaching mathematics at Robert B. Glenn High School where she taught for 11 consecutive years. She taught numerous courses during her time as a high school teacher including Algebra I and II, Geometry, Integrated Math II, Advanced Functions and Modeling, Pre-Calculus, and AP Calculus AB.

Tracy entered graduate school at Wake Forest University immediately after her first year teaching. She earned a Master of Arts in Education with a mathematics concentration in December 2002 and worked on the clinical staff at WFU for a few years. Fall 2003, was the start of eight years of service as an adjunct instructor in the mathematics department at WSSU. Tracy has also taught mathematics in many informal learning environments, and she has devoted the last 10 years to teaching middle and high school students in the NC-Mathematics, Science, and Education Network at WSSU. After a little over a decade's worth of teaching experiences, she decided to embark upon her own scholastic aspirations and applied to North Carolina State University's doctoral program in Mathematics Education. Tracy worked as a teaching assistant for Introduction to 21st Century Teaching, an undergraduate education course, for two years of her doctoral program. She also spent two and a half years as a research assistant for Project All Included in Mathematics (AIM), a National Science Foundation (NSF) funded project. From this work, Tracy has submitted publications to journals and presented at numerous conferences. She has been a member of various panel discussions in the College of Education at NC State and has worked for three years as a graduate student representative on the College of Education's Committee on Multicultural Issues and Diversity (COMID). As a PhD Graduand, she taught two sections of Classroom Assessment Principles and Practices, an upper level undergraduate education course at NC State University.

Tracy's goal is to obtain a tenure track faculty position in mathematics education where she can teach methods courses as well as mathematics courses on the undergraduate and graduate levels. She also plans to continue pursuing her research interests and wants to explore grant writing.

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I am forever grateful for my parents. They have always encouraged me to be the best me that I could be and to follow my dreams. They taught me to treat people with respect, accept people as unique individuals, and to "stand for something" at a very young age. Thanks for supporting my decision to not be in academically gifted (AG) classes in elementary school when many thought that you should not have let me make that decision. Additionally, I thank you for supporting my decision to attend Winston Salem State University, the local Historically Black College and University, when people thought I was "too smart for that school." Despite the naysayers, I did it!!!

Thanks to my family for always believing in me and for showing me what love and family mean. Special thanks to my grandmother and my aunt Luzelle for all that you do for my mom; because of you I felt comfortable leaving Winston Salem after my dad passed to embark on this journey. Thanks to my friends Evette, Rhonda, Talia, Britt, Shetika, and Tisa (and those unnamed) for their patience and support because I have often bored them with my lists of things to do and graduate school struggles. Thanks to my in-laws for being so loving and supportive. Thanks to my former teachers, coworkers, and students. Thanks to all of the people that I have built relationships with at NC State. I would be remiss if I did not personally thank my aunt Julia; it is because of her that I decided at the tender age of six that I wanted to be an educator.

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CHAPTER 1: INTRODUCTION

The importance of discourse as a tool for teaching and learning mathematics has been recognized for decades in K-12 mathematics education (Klein, 2007; NCTM, 1989; 1991; 2000; NGACBP & CCSSO, 2010). More recently, discourse has become a focus in undergraduate mathematics teaching (VanCleave & Fredericks, 2009). Discourse holds an important place in mathematics teaching because it is *capable* of enhancing mathematics learning (Hiebert & Wearne, 1993; Huang, Normandia, & Greer, 2005). The operative word here is *capable* as discourse, even that specific to mathematics, does not necessarily result in deep understanding of mathematics (Higgins, Cermak-Rudolf, & Blanke, 2009; Kazemi & Stipek, 2001; Sherin, Mendez, & Louis, 2000). Research in K-12 mathematics classrooms has demonstrated that *effective discourse* (i.e. discourse that impacts understanding) includes students conjecturing, collaborating with their peers as they think through mathematical tasks, explaining, and justifying their thoughts (Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Higgins et al., 2009). Further, this body of research has emphasized that mathematics teachers' instructional practices, especially questioning, impact the quantity and quality of students' mathematical discourse experiences (Kazemi & Stipek, 2001).

K-16 mathematics teaching in the United States has often been branded as "traditional" (Franke, Kazemi, & Battey, 2007; Hiebert & Stigler, 2000) despite undeniable efforts to change this connotation (Asthagiri, Brezhneva, Byrkett, Farmer, Harper, Keeler, & Anderson, n.d.; Blair, Kirkman, & Maxwell, 2013; Higgins et al., 2009; Rasmussen, Kwon, Allen, Marrongelle, & Burtch, 2006; Yoshinobu & Jones, 2013). Traditional instruction is a teacher-centered approach that utilizes lecture as the primary means of teaching mathematics

(Prawat, 1992; Wood, Cobb & Yackel, 1991). Student-teacher interactions are commonly restricted to Initiate Respond Evaluate (IRE) patterns (Mehan, 1979), a sort of call and response, which only allows students to communicate short responses with anticipation of validation from the teacher (Desimone, Smith, Baker, & Ueno, 2005; Hiebert & Stigler, 2000). The issue with traditional instruction is not that it utilizes lecture or displays IRE patterns; it is that these practices are used almost exclusively. Extreme use of such practices limit students' opportunities to engage in mathematical thinking because it constricts the time they have to talk about their thinking and to listen to the thoughts of others (Franke, Fennema, & Carpenter, 1997). One practice that has the possibility of changing this is questioning.

Students encounter instructors' questions in various formats including mathematical tasks to work on, exercises to complete, and the questions they ask verbally (Brodie, 2009). Each format holds its own importance, but instructors' verbal questions are of interest because they open the door for discourse and guide where it goes. Instructors' verbal questions can also impact the other formats stated. For example, Henningsen and Smith (1997) reported that the questions instructors ask can decrease the cognitive demand of mathematical tasks. The questions instructors ask students while engaged in mathematical conversations can either support or restrict students' opportunities for deeper mathematical understanding, therefore it is important to be cognizant of how instructors use their questions with students. The focus of this study will be instructor questioning in undergraduate mathematics, however, the setting is Calculus I classes.

Undergraduate Calculus I has been of interest in the field of mathematics education for more than 25 years. An impetus in sparking this attention was the National Science Board's (1986) Neal Report which described issues in undergraduate engineering, mathematics, and science (Ganter, 1999). Low retention rates and less than stellar student performances are attributed to a need for change and resulted in the *calculus reform movement*. The National Science Foundation contributed greatly to the movement by granting numerous institutions with projects between 1988 and 1994 (Ganter, 1999). As a result of the *calculus reform movement*, institutions nationwide made changes in alignment with at least one of the following themes: constructivism, cooperative learning, 'real world' problems and communication, syllabus revision, and technology integration (Robert & Speer, 2001). Additionally, the 'Rule of Three' which places emphasis on integrating graphical, numerical, and algebraic representations was deemed integral to teaching calculus (Hughes-Hallett et al, 1994). The changes impacted the curriculum as well as instructional practices.

The *calculus reform movement* contributed to an increase in research interests around calculus students. Many studies were comparative studies due to the fact that calculus reform projects focused on alternatives to traditional instruction (Robert & Speer, 2001). The results of these studies indicate that there are both cognitive and affective gains associated with teaching Calculus I in a more student-centered way. Reform Calculus I classes have proven to change students' beliefs on the value of mathematics, increase their confidence in their ability to do mathematics, and alter their attitudes towards it (Bookman & Friedman, 1998; Laursen et al., 2011). Students have seen improved grades, increased conceptual understanding, and persistence in mathematics courses (Johnson, 1995; Schwingendorf, 1999; Treisman, 1992). These gains are all relative to the prior knowledge of students in reform calculus courses, their previous thoughts and feelings, and/or students in traditional calculus courses.

Similar to the comparative studies described above whose purpose was to establish that there is undeniable value in reform calculus for students, much of research involving Calculus I has focused heavily on students. Researchers have studied students' understanding of various concepts including derivatives, limits, functions, volume, and related rates (Dorko & Speer, 2013; Martin, 2000; Orton, 1983a; 1983b; Szydlik, 2000). Additionally, experts have discussed the concepts and skills that are important for students to learn in Calculus I and have established four end goals for students' understanding of it: (a) mastery of the fundamental concepts and-or skills of the first-year calculus, (b) construction of connections and relationships between and among concepts and skills, (c) the ability to use the ideas of the first-year calculus, and (d) a deep sense of the context and purpose of the calculus. (Sofronas et al., 2011, p. 134). Learning theories have also been of interest (Asiala, Cottrill, Dubinsky, & Schwingendorf, 1997), yet minimal attention has been devoted to the instructors of calculus and their teaching practice (Speer, Smith, & Horvath, 2010). This holds true despite open acknowledgement of changes in how the course is taught.

Nigam (1998) is one of the few that has tackled the task of researching teaching in calculus courses and examined the role of visualization in teaching undergraduate mathematics. The researcher observed Precalculus, Calculus I, Calculus III, and Calculus IV classes and found that graphing, geometrizing, physical modeling situations, and visually evocative language were the main visual teaching strategies utilized. Of the four practices identified, graphing was used most often. Little research has been done on discourse or more specifically questioning. One exception is Roach, Roberson, Tsay, and Hauk (2010) who explored the questioning strategies of Calculus I instructors which will be discussed in detail in the next following chapter. The current study also examines Calculus I instructors' questions, yet the participants were full-time mathematics instructors at the time of the study opposed to graduate teaching assistants as in Roach et al.'s (2010) work.

Purpose of the Study

This study aims to contribute to what is known about the ways instructors use their mathematical questions in undergraduate Calculus I classes. There is a focus on undergraduate classes because of the need to change instruction at this level to a more student-centered model where discourse is a core component (Blair et al., 2013). Calculus I was selected as the mathematics course for examining instructors' questioning for many reasons. First, Calculus I is an introductory course for Science, Technology, Engineering, and Mathematics (STEM) majors and is a course that has been shown to influence retention in these majors. Seymour and Hewitt (1997) found that students who switch out of STEM fields of study often do so because of poor teaching that lacks interactions between the instructor and the students. It has also been reported that one introductory class can impact students staying in STEM majors (Ellis, Rasmussen, & Duncan, 2013; Watkins & Mazur, 2013). This makes Calculus I the operative course to study instructors' questioning because it is a context that needs attention if we want to maintain or possibly gain STEM majors. Calculus I is also a requirement for many majors outside of STEM (e.g., business, economics). Pre-service mathematics teachers are also often students in Calculus I. Mathematics courses for these students are not just about mathematics because their instructors stand as models of ‘how to teach.’ Given that STEM and non-STEM majors together produce a multitude of students who have to take this course suggests that what goes on during class time should not be taken lightly.

With a goal of participating in undergraduate mathematics reform, it is necessary to understand the ways instructors are using mathematical questions during class time. This study has one encompassing goal; provide a description of the ways undergraduate Calculus I instructors are using their mathematical questions during class time. It seeks to answer the following question:

> How are mathematics instructors using their mathematical questions in undergraduate Calculus I classes?

Specifically:

a. What types of questions are asked by Calculus I instructors?

b. In what ways do instructors utilize wait time with their mathematical questions?

c. In what classroom setting(s) are the various question types and wait time observed?

Definitions of Terms

This section will discuss definitions of terms that may be defined differently in K-16 mathematics education research. I will begin by defining discourse generally before drawing attention to discourse in the context of mathematics classrooms and wait time.

discourse. Gee (2012) described little "d" discourse as "language in use or connected stretches of language that make sense, like conversations, stories, reports, arguments, essays,…" (p. 151).

Mathematical discourse. "Language, spoken or written, regarding mathematics" (Stolk, 2013, pp.7-8). In this study there will be an emphasis on mathematical discourse that is spoken. Since the study is situated in mathematics classrooms, the terms discourse and mathematical discourse will be used synonymously.

Discourse. Big "D" discourse is "composed of distinctive ways of speaking/listening and…coupled with distinctive ways of acting, interacting, valuing, …thinking, believing, with other people and with other objects, tools, and technologies, so as to enact specific socially recognizable identities engaged in specific socially recognizable activities" (Gee, 2012, p. 152).

Mathematical Discourse. "Ways of representing, thinking, talking, agreeing, and disagreeing about mathematics" (Stolk, 2013, p. 7). In this study Discourse will be used synonymously with mathematical Discourse because Calculus I classes are the context.

Wait time. The gap of time between an instructor's question and the initial utterance in the following response (Rowe, 1986).

Significance of the Study

This qualitative collective case study is significant for many reasons. First, the study is noteworthy because it investigated an under-examined teaching practice at the undergraduate level (Speer, Smith, & Horvath, 2010). Boaler and Brodie's (2004) categories of questions informed the conceptual framework and were used for analyzing data in regard to question types. Their categories offer a valuable lens for analyzing instructors' mathematical questions, yet they were established based on their observation of secondary mathematics teachers. I adapted Boaler and Brodie's (2004) categories of questions such that they are appropriate to use in Calculus I classrooms. The question types presented offer a tool for instructors' reflection and growth, and may be appropriate for use in other undergraduate mathematics courses. This study also contributes to the growing body of empirical research in undergraduate mathematics education. Lastly, this research holds value in professional development for undergraduate mathematics instructors because it provides real life examples of instructors' teaching practice and offers foundational tools for exploring questioning.

Overview of Methodological Approach

Qualitative research is an inquiry approach conducive to the in-depth study of issues, people, and phenomena (Creswell, 2013; Patton, 2002). Research of this kind is situated, making the researcher a data collection instrument in a pool of naturally occurring events (Creswell, 2013; Denzin & Lincoln, 2000; Miles, Huberman, & Saldaña, 2014). This approach allows researchers to paint holistic images of multifaceted entities using rich data provided by purposefully selected samples (Creswell, 2013; Patton, 2002). Open-ended questions are a common feature of qualitative research (Creswell, 2013; Denzin & Lincoln, 2000; Patton, 2002). Utilizing a qualitative approach in this study allowed me to explore instructors' questioning in an undergraduate mathematics course, specifically, Calculus I.

Qualitative methods have a dominant presence in research on discourse in K-12 mathematics classrooms (Ryve, 2011) suggesting their worth for exploring questioning, a core element of discourse, in undergraduate mathematics classrooms.

Collective case study was selected as the qualitative approach in this study. The participants in this study came from five different Calculus I classes. The instructors were selected based on their willingness to participate and proximity to the instructor. Student participants were enrolled in the selected instructors' Calculus I classes. Each instructor participated in a semi-structured, audio-recorded, interview as well as classroom observations. Multiple hours of in-class observations were conducted; both audio- and video-recorded. Verbatim transcripts of all observations were produced. Interview data was used to create instructor profiles, and transcripts from observation data were analyzed using a conceptual framework informed by literature that will be described in detail later.

Chapter Summary and Organization

This study is organized in five chapters. The first chapter introduced the background, research questions, and rationale, and provides an overview of the study. Chapter Two presents a synthesis of relevant literature in K-16 mathematics. Special attention is given to teacher questioning. It is believed that this literature can be used to further inform research on instructors' questioning at the undergraduate level. The third chapter delivers a description of the research design, research questions, data collection, and data analysis. Additionally, validity and reliability strategies, ethical issues, and researcher subjectivity are presented. Chapter Four is devoted to the findings of the study which are discussed in detail first by case and then through cross-case analysis. The final chapter presents the answers to the research questions with connections to literature, implications for the field, limitations of the study, and potential future research.

CHAPTER 2: LITERATURE REVIEW

In order to understand how undergraduate mathematics instructors are using the mathematical questions they ask their students, one must be aware of different kinds of questions instructors ask and what they are giving students opportunities to engage in by asking them. This chapter includes a quick overview of research relating to the value of questioning, yet the main section is devoted to detailing how instructors' questions have been discussed and categorized in research. Wait time is discussed because it is essential to understanding how instructors use their questions. Additionally, the classroom setting during mathematics instruction is included because it is there that instructors' questions are posed.

The Value of Questioning

There is no doubt that instructors' questioning is of consequence in mathematics teaching and learning because it stands as a core element of discourse in the classroom (Blair et al., 2013; Klein, 2007; NCTM, 1989; 1991; 2000; NGACBP & CCSSO, 2010). Just as all talk about mathematics is not associated with students' understanding of mathematics, all instructor questions do not support it (Hufferd-Ackles, Fuson, & Sherin, 2004; Martino & Maher, 1999). Research has separated mathematics instructors' questioning into two general categories (Hiebert & Wearne, 1993; Woods, 1998). The first category requests that students recall procedures or state factual information, and the instructor's focus is on the correctness of answers (Hufferd-Ackles et al., 2004; Roach et al., 2010; Sorto, McCabe, Warshauer, & Warshauer, 2009). The second provides students with opportunities to make their understanding explicit, it develops students' critical thinking, and it increases student participation (Aizikovitsh-Udi & Star, 2011; Martino & Maher, 1999; Mesa, 2010).

When mathematics teachers pose questions aimed at engaging students in understanding mathematics, they not only give students a chance to verbalize their thoughts and/or reasoning, but give themselves access to students' thoughts that may not have otherwise been shared (Martino & Maher, 1999; Roach, Roberson, Tsay, & Hauk, 2010). Knowing what students are thinking makes it easier for instructors to make informed instructional decisions (Franke et al., 2009). This kind of questioning also produces shared responsibility for learning because the teacher is no longer viewed as the owner of knowledge. Students are exposed to each other's thoughts which allows them make connections or reevaluate their own understanding.

It is important to note that questioning recognized as procedural and less engaging has a place in mathematics classrooms depending on how instructors use it (Boaler & Brodie, 2004; Temple & Doerr, 2012). Questions that focus on recall or factual answers can in fact be important for setting the stage for learning new mathematics (Boaler & Brodie, 2004; Roach, Roberson, Tsay, & Hauk, 2010; Stolk, 2013). They shed light on students' prior knowledge and experiences and are capable of bringing in relevant context. These questions should not, however, be the dominant form of questions used.

Aside from knowing the impact of questioning, one needs to know what types of questions lie inside what may be viewed as overarching categories. The next section presents an overview of literature that informs what is known about mathematics instructors' questions based on how they have been categorized.

Categorizing Instructors' Questions

Numerous researchers have given attention to instructors' questioning in mathematics classrooms and left behind categories for characterizing the questions asked. Instructors' questions have often been categorized by pattern, level, and/or type and have been described based on their ability to engage students in discourse that promotes their conceptual understanding of mathematical ideas. In this section I synthesize literature around the common categorizations of instructors' questions and introduce the framework that will be utilized in this study.

Patterns of Questions

The patterns observed in mathematics instructors' questions help to identify the questioning practices instructors use "to help make student thinking explicit" (Franke et al., 2009, p. 383). One cannot ignore the commonly observed IRE patterns of Mehan (1979) where the teacher asks a short-response question that focuses on a correct answer, gets a student response, and evaluates its correctness. Alternative patterns have been explored due in part to the ineffectiveness of IRE questioning patterns in promoting students' thinking and articulation of it (Callard, 2009). Wood (1998) highlighted two distinct questioning patterns, funneling and focusing. Funneling is a pattern where instructors use their questions to guide students to correct answers. This pattern is similar to IRE patterns. Contrarily, focusing patterns involve instructors asking students questions that concentrate on students' explanations without drawing attention to a pre-specified direction or answer.

Researchers have used Wood's (1998) questioning patterns to describe how instructors use their questions in mathematics classes to elicit students thought, but one will find that there is more mention of funneling than focusing as it has been the commonly observed pattern (Aizikovitsch-Udi & Star, 2011; Franke et al., 2009; Fukawa-Connelly, 2012; Roach et al., 2010; Sorto et al., 2009; Temple & Doerr, 2012). Fukawa-Connelly (2012) found that an undergraduate abstract algebra instructor used a funneling pattern when teaching proofs. The researcher provided excerpts that detailed the instructor asking factual questions. At times the instructor asked higher cognitive level questions followed by a series of factual questions.

Temple & Doerr (2012) explored the interactional strategies that a tenth grade mathematics teacher used to develop her students' facility with the mathematics register. Unlike Fukawa-Connelly (2012) who only found evidence of funneling patterns, Temple and Doerr (2012) observed both funneling and focusing. It was established that the observed teacher used funneling patterns when her goal was for students to talk about previously learned content. On the other hand, the teacher implemented focusing patterns when the objective was for students to co-construct new ideas. The researchers compared funneling and focusing to Franke et al.'s (2009) leading and probing.

Franke et al. (2009) observed four "types" of teacher questioning patterns in elementary classrooms that teachers used to build on students thoughts; general questions, specific questions, probing sequences of specific questions, and leading questions. General questions and specific questions are opposites in that the former is not directly related to anything a student has said but the latter is. Probing sequences of specific questions involves at least two or more related questions of the specific type, yielding multiple teacher questions and multiple student responses. This pattern can be compared to focusing because teachers are digging into students' thoughts by asking questions that utilize their explanations when probing. Leading has been deemed synonymous with funneling as the teacher asks students questions that guide them toward specific answers or explanations.

Callard (2009) studied the teacher questioning patterns in his 7th and 8th grade mathematics classrooms and found that his persistent pattern was similar to the IRE patterns of Mehan (1979). The teacher-researcher indicated that his patterns regularly began with an inquiry, got a student response, and was followed with some sort of cue indicative of his evaluation of the student's response. Callard (2009) also called this the IRE pattern, but the "I" was for inquiry. In addition to IRE patterns, the researcher found that he limited students' opportunities to engage by utilizing a unilateral approach.

Callard (2009) combined the descriptions of the commonly observed patterns in his class and came up with the Unilateral Inquiry Response Evaluation (UIRE) pattern. Further, the research established three additional questioning patterns including Multilateral Inquiry Response Evaluation (MIRE), Inquiry Response Collection (IRC), and Inquiry Response Revoicing Controversy (IRRC). The MIRE pattern is similar to the UIRE pattern in that the teacher begins with a question, a student answers, and the teacher evaluates the answer. The difference is multiple students are allowed to answer the same question presenting chances for more students to engage. Under the IRC patterns, the teacher asks open-ended questions that do not have one correct answer and collects responses from multiple students without verifying or validating their answers. Lastly, the IRRC pattern involves dealing with students' misconceptions using revoicing with questioning. All students are challenged to participate and justify their arguments, and cognitive and metacognitive benefits were reported. The researcher pointed out that both leading and probing questions were asked suggesting that there was a sense of funneling and focusing on the teacher's behalf.

Some questioning patterns have been deemed more effective than others, in regard to engaging students at higher cognitive levels, but they do not tell us much about the questions within the patterns aside from them being open or closed. Probing questions for example have been identified as having potential benefits for students, but instructors can ask varying types of questions that probe and the questions can probe on different levels (Franke et al., 2008). It cannot be assumed that when a mathematics teacher or instructor poses open-ended questions they produce the same results.

Levels of Questions

Instructors' questions have been discussed across various levels offering different views on how levels themselves are defined (Emerson, 2010; Hufferd-Ackles et al., 2004; Marshall, Horton, & White, 2009; Mesa, 2010; Mills, 2013; Roach et al., 2010; Smart & Marshall, 2013). One of the most familiar frameworks involving levels is Bloom's Taxonomy (Anderson et al., 2001; Bloom, 1956). The original taxonomy included the following levels from lower to higher order: knowledge, comprehension, application, analysis, synthesis, and evaluation (Bloom, 1956). The modified taxonomy changed the part of speech from noun to verb form and switched the top two levels yielding the following levels: remembering, understanding, applying, analyzing, evaluating, and creating (Anderson et al., 2001).

Emerson (2010) used the top three levels of Anderson et al.'s (2001) revised Bloom's Taxonomy to frame his study on the connections 8th grade Geometry students make when higher-level questions are posed by their teachers. On the other hand, Mesa (2010) and Mills (2013) gave attention to all of the modified Bloom's levels in their work with undergraduate instructors. Mesa (2010) used Bloom's Taxonomy to classify the questions of seven community college mathematics instructors by the cognitive process they required. Similarly, Mills (2013) used the taxonomy to gauge the level of cognitive engagement offered by four undergraduate advanced mathematics instructors' questions. Mesa (2010) further classified questions based on if instructors ever intended for students to respond, and Mills (2013) included the expected response type of the instructors' questions. The additional question types offered will be discussed in a later section.

Hufferd-Ackles et al. (2004) established four levels of math talk, across four components of discourse, in their research conducted in a third grade classroom. Questioning was identified as one of the four components of discourse and was discussed from Level 0 to Level 3. The shift through levels entails the teacher moving from the sole questioner to evidence of the teacher and students as questioners. Further, the teachers' questions shift from focusing on answers to having students think deeply about their explanations and strategies.

In Hufferd-Ackles et al.'s (2004) work, Level 0 questions focus on answers and give little to no attention to eliciting students' thoughts. At Level 1, teacher questions begin to focus on students' thinking, but there are still questions that focus on answers. Teachers may also ask follow-up questions at Level 1 providing evidence of slight probing. Teachers probe deeper at Level 2 and asks questions that are more open in nature. At the highest level, Level 3, there is an emphasis on students asking each other questions. Here the teachers' questioning is minimized, but she/he is still engaged in the math talk. Hufferd-Ackles et al. (2004) reported that classes move through the levels of math talk in a linear fashion, but when new topics are introduced the trajectory starts over with a swift progression back to Level 3.

Similar to Hufferd-Ackles et al. (2004), Marshall et al. (2009) used four levels to measure classroom discourse. Discourse was one of five constructs discussed in their instrument designed to measure the quantity and quality of inquiry-based instruction in mathematics and science classes. Their levels of discourse were identified as Pre-Inquiry, Developing Inquiry, Proficient Inquiry, and Exemplary Inquiry; Levels 1-4 respectively. The discourse aspect entailed three sub-constructs related to questioning, namely, questioning level, complexity of questioning, and questioning ecology. Communication patterns and classrooms interactions were also a part of the discourse construct.

Marshall et al.'s (2009) questioning levels relate to the modified Bloom's Taxonomy as the questions progress from levels that do not exceed remembering to those that include analysis or higher. Their complexity of questions is similar to Hufferd-Ackles et al.'s (2004) work because the emphasis is on teachers' use of questions that have one correct answer or open responses with attention to students' thinking. Questioning ecology was included in order to provide a glimpse of teachers' success in engaging students. Instructors who lecture or engage students in questions that are of low cognitive level, for example, are at Level 1 because they are not producing discussion. The communication patterns sub-construct allows one to document who controlled or guided the conversation, whereas classroom interactions focuses on the use follow-up questions and probes. For example, the Pre-Inquiry level of communication patterns states, "Communication was controlled and directed by teacher and followed a didactic pattern" (Marshall et al., 2009, p. 49). The Pre-Inquiry level of classroom interactions indicates that the teacher accepts students' answers, sometimes correcting them, but rarely follows up with probing.

Sorto et al. (2009) considered levels from a different perspective. The researchers investigated 6th and 7th grade mathematics teachers' questions on three "levels" of analysis; questions in isolation, questions in their neighborhoods, and questions in the overall lesson. Their analysis of questions in isolation looked at the level, type, and pattern of individual questions using previously established models found in literature. This level of analysis is similar to the levels discussed above in that it tends to the cognitive level of questions, the kind of questions (e.g., open versus closed questions), and the focus of the teachers questions (e.g., answers or students' ideas and thoughts). Questions in their neighborhoods displayed the context the questions were posed in by looking at the conversation that occurred around the teachers' questions. In a sense, this allows the teachers' intentions to be captured. Questions were examined in the overall lesson to provide the flow of the lesson.

Roach et al. (2010) used Hufferd-Ackles et al. (2004) and Sorto et al. (2009) to frame instructors' mathematical questioning Discourse. Roach and colleagues used Hufferd-Ackles et al.'s (2004) math talk levels to establish what they called college instructors' levels of question-related discourse. The researchers first coded for question type using codes from their previous work specifically, comprehension check, eliciting, and probing. The instructors' questioning level was then documented every 10 minutes adding the instructors' intent and the cognitive demand required of the question.

The levels discussed offer different researchers views on how questions can be classified. Hufferd-Ackles et al. (2004) and Marshall et al. (2009) suggest that instructors' questioning progresses through levels that represent their skill in implementing discourse in mathematics classes. A drawback with frameworks of this nature is that they suggest an upward progression, yet instructors' questions and the resulting discourse do not always follow a linear progression. Each framework detailed gave attention to cognitive level or cognitive engagement. Further, there was at minimum mention of questioning patterns like probing and funneling.

Question Types

Similar to research on question levels and question patterns, questions types have generally been provided in broad categories that demonstrate mathematics teachers' efforts to elicit students' thoughts and participation (Mesa, 2010; Mills, 2013; Rasmussen, Kwon, & Marrongelle, 2008; Sahin & Kulm, 2008). Sahin and Kulm (2008) found that two sixth grade mathematics teachers displayed three question types in their classes: probing, guiding, and factual. Probing questions were identified as asking students for explanation or elaboration of their thinking, application of their prior knowledge, or justification of their ideas. Guiding questions were characterized as asking for specific answers or next steps in solving problems, and are similar to Franke et al.'s (2009) leading questions in that they show up as sequences of factual questions that lead students to particular answers to and approaches for solving problems. Sahin and Kulm (2008) stated that factual questions and guiding questions are sometimes connected because of the attention that factual questions give to specific answers, next steps of procedures, facts, and definitions.

Rasmussen et al.'s (2008) Inquiry-Oriented Discursive Move (IO-DM) framework included questioning/requesting as one of four teacher discursive moves observed in an undergraduate Inquiry-Oriented Differential Equations (IO-DE) course. Their questioning/requesting category consists of four sub-categories; evaluating, clarifying, explaining, and justifying. The researchers pointed out that the subcategories reflect a general advancement in mathematical intricacy from declarative knowledge to relational knowledge indicating that lower level and higher level questions are present. Rasmussen and colleagues' (2008) IO-DM framework went beyond establishing question types and offered connections between the question types and their functions of teacher and student inquiry.

Teachers are able to model their students' thinking, learn new mathematics, and pose new questions and tasks when they inquire into their students' mathematical thinking (Rasmussen et al., 2008). Through inquiry, students are provided opportunities to engage in learning new mathematics using argumentation. Rasmussen and Kwon (2007) explained that student inquiry also functions to "empower learners to see themselves as capable of reinventing mathematics and to see mathematics itself as a human activity" (as cited in Rasmussen et al., 2008, p. 2). The authors pointed out that all question types do not serve all of the listed functions, and different types of questions attend to different functions.

The explaining sub-category was identified as the only question type that tends to each function of teacher and student inquiry.

Mills (2013) offered seven types of questions undergraduate mathematics instructors ask based on the expected response type: comprehension, rhetorical, choice, product, process, and meta-process. Comprehension questions are those that check for student understanding. As such, instructors pause for more than two seconds so that students are given a chance to respond. Rhetorical questions do not seek anything from students as they are the questions instructors never expected to have answered. Choice questions force students to agree or disagree with a statement. Product questions are similar to what has been called factual questions (Sahin & Kulm, 2008) in that they draw attention to factual responses. The last two question types offered by Mills (2013) align with the explaining and justifying sub-categories in the IO-DM framework (Rasmussen et al., 2008). Further, they fit into what Sahin and Kulm (2008) called probing questions. Process questions focus on students' thoughts and ideas, while meta-process questions draw students' attention to their thinking and request justification of their thoughts (Mills, 2013).

Thus far, an overview of research that gives attention to categories of questions that mathematics instructors' ask during class has been provided. It has been established that there are overlying similarities in researchers' categorizations of mathematics instructors' questions despite their organization by levels (Emerson, 2010; Hufferd-Ackles et al., 2004; Mills, 2013; Roach, Roberson, Tsay, & Hauk, 2010; Smart & Marshall, 2013), patterns (Callard, 2009; Fukawa-Connelly, 2012; Mehan, 1979; Temple & Doerr, 2012; Wood, 1998), and types (Mesa, 2010; Mills, 2013; Rasmussen et al., 2008; Sahin & Kulm, 2008).

Each classification has highlighted questions that are separated by a line of skill which has questions that focus on remembering, facts, answers and the like on one side, and questions that elicit student thinking and justifications on the other. This can be seen with Bloom's Taxonomy where the bottom two tiers and the top three tiers have been separated to represent low-level versus high-level questions. The grouping of questions has typically been painted with broad strokes creating a need for research that offers more detail.

Categorizing Instructors' Questions with Smaller Pieces

Boaler and Brodie (2004) presented a more thorough image of the types of questions instructors pose with nine categories of questions as seen in Table 1. The researchers observed six secondary mathematics classes with different instructors and found that each of the instructors asked Type 1 (Gathering information, leading students through a method) questions with the highest frequency. The instructors who taught from traditional curricula had higher percentages of this question type (95% or higher) compared to reform teachers (between 60 and 75%), and reform teachers asked a wider range of questions. Question types 3 and 5 were only observed by reform teachers, yet the question types made up varying percentages of the instructors' questions. The former focuses on conceptual understanding while the latter seeks to get students engaged in discussions. On the other hand, Type 4 was asked by most instructors suggesting that mathematics teachers' probed into their students' thinking.

Table 1: Boaler and Brodie's (2004) Categories of Teacher Questions

Question type	Description	Examples
1. Gathering information, leading students through a method	Requires immediate answer Rehearses known facts/procedures Enables students to state facts/procedures	What is the value of x in this equation? How would you plot that point?
2. Inserting terminology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them	What is this called? How would we write this correctly?
3. Exploring mathematical meanings and/or relationships	Points to underlying mathematical relationships and meanings. Makes links between mathematical ideas and representations	Where is this x on the diagram? What does probability mean?
4. Probing, getting students to explain their thinking	Asks student to articulate, elaborate or clarify ideas	How did you get 10? Can you explain your idea?
5. Generating Discussion	Solicits contributions from other members of class.	Is there another opinion about this? What did you say, Justin?
6. Linking and applying	Points to relationships among mathematical ideas and mathematics and other areas of study/life	In what other situations could you apply this? Where else have we used this?
7. Extending thinking	Extends the situation under discussion to other situations where similar ideas may be used	Would this work with other numbers?
8. Orienting and focusing	Helps students to focus on key elements or aspects of the situation in order to enable problem-solving	What is the problem asking you? What is important about this?
9. Establishing context	Talks about issues outside of math in order to enable links to be made with mathematics	What is the lottery? How old do you have to be to play the lottery?

Stolk (2013) described seven of Boaler and Brodie's (2004) question types as "skillful in eliciting student thinking or engaging students in developing mathematical understanding" (p. 12). She pointed out the following question types: Inserting terminology, Exploring mathematical meanings and/or relationships, Probing or getting students to explain their thinking, Linking and applying, Extending thinking, Orienting and focusing, and Establishing context. Stolk (2013) expounded upon Boaler and Brodie's (2004) work to establish the types of questions that teachers ask in classes that are specifically conceptually-driven. She organized her work in five broad categories further arranged into 21 subcategories, namely, Accessing Relevant Information, Exploring the Mathematics, Explaining One's Thinking, Analyzing Explanations, and Linking and Applying. Unlike Boaler and Brodie (2004), Stolk (2013) observed one undergraduate mathematics class for pre-service teachers and found that the instructor asked conceptual questions with the highest frequency.

Hähkiöniemi (2013) sought to elaborate on different ways of asking students to explain during mathematics lessons. He highlighted three of Boaler and Brodie's (2004) question types as asking students to explain, specifically, Exploring mathematical meaning and/or relationships, Probing or getting students to explain their thinking, and Extending thinking. The researcher gave particular attention to probing and established seven types of probing by observing preservice teachers implement an inquiry-based mathematics lesson to secondary students. Hähkiöniemi (2013) presented Probing method, Probing reasoning, Probing cause, Probing meaning, Probing argument, Probing extension, and Unfocused probing suggesting that teachers ask students to explain different things.

Dong, Seah, and Clarke (2015) built upon Boaler and Brodie's (2004) work to explore teachers' questioning practice in reform-based mathematics curriculum. They analyzed instructors' initiation-response-follow up (IRF) exchanges (Cazden, 2001) using two categories of questions; initiation and follow-up. Eleven sub-categories of initiation questions were described (e.g., *Evaluation, Link/application, Result/product, Comparison,* and *Information extraction*), each of which serves to start a conversation. Nine sub-categories intended to follow-up on students' contributions were described including *Clarification, Justification, Cueing,* and *Agreement request.* The researchers found that each of the initiation and follow-up question sub-categories were put to use by the instructor, however, some were used more regularly than others. The instructor tended to initiate discussions with *Understanding check, Review,* and *Explanation request* questions, and generally followed-up with *Elaboration* questions.

Stein and Smith (2011) identified asking good questions as one of five practices for orchestrating productive mathematics discussions. Their work highlights Boaler and Brodie's (2004) question types as a means of examining questioning in K-12 mathematics classrooms and points out three question types (Probing or getting students to explain their thinking, Exploring mathematical meaning and/or relationships, and Generating discussion) of particular importance. I aimed to establish that Boaler and Brodie's (2004) categorization of questions is appropriate for understanding the questioning of multiple undergraduate instructors, noting that new categories or subcategories of questions may arise. The categories tell us about the questions instructors ask, but in order to understand the ways they are using their questions attention must be given to the time they wait for students to answer them.

Wait Time

When instructors ask questions they either pause for students to respond or they do not. The time that instructors pause between their question and the start of a student response has been identified as wait time, and it during this time that students are given an opportunity to think and develop their responses (Rowe, 1986). Wait time is an integral aspect of instructors' questioning that can positively impact both the students and the instructor if allocated appropriately (Kwit, 2012; Rowe, 1986; Tobin, 1987). It is however, a technique that is often forgotten when questioning takes place (McComas & Abraham, 2004). Wait time should not be overlooked when attention is drawn to instructors' questioning because it does not matter how good the instructors questions are, if students are not given a fair shot at being able to answer them.

Different types of wait time have been detailed in literature. Rowe (1986) established three types of wait time. Wait Time 1 is the time between an instructors' question and when a student begins to respond. Wait Time 2 to be the time between when a students' response ends and when the teacher starts to talk again. She also discussed a modified wait time for classes that are primarily lecture. This particular kind of wait time incorporates cycles of eight to 10 minutes of lecture followed by a two minute break where students work with each other to clarify concepts. More recently, Heinze and Erhard (2006) detailed an additional type of wait time called Wait Time 3. The researchers pointed out that teachers often stop students to say something after they have asked a question, creating a gap in the original wait time. This type of wait time is not captured all the time because instructors do not always have something additional to say. The time between what instructors add and when a student answers the question is Wait Time 3. In this study wait time will be the time between an instructor asking a question and the next person's response. The next person could be either the instructor or a student.

How Long Should Wait Time Be?

K-12 teachers and undergraduate instructors alike tend to give limited wait time after asking questions. It has been reported that instructors' average wait time once a question has been posed is around one second (Aizikovitsh-Udi & Star, 2011; Kwit, 2012; McComas & Abraham, 2004; Rowe, 1986). An average this low is startling when instructors' questions stand as a primary means of engaging students during class time (Sharill, 2013). According to Mesa (2010), pauses under two seconds constitute no wait time at all. This is reasonable because it could take students that long to process what is being asked of them. Mesa considered wait time to be pauses of three seconds or more allowing one to extract that pauses between two to three seconds could be considered minimal wait time.

Higher average instructor wait times have been conveyed in literature, yet they remain below the three second mark (Duell, 1994; Mesa, 2010). This may be of concern for some as Rowe (1986) stated that "There is a threshold value below which changes in wait time produce little effect and above which (2.7 seconds) there are marked consequences for both teachers and students" (p. 43). It should not be concluded that all wait times need to be over the threshold value because there are questions that students tend to respond to immediately (Mesa, 2010; Stolk, 2013). For example, "What did you get for number eight?"

Higher level questions have, however, been associated with wait times of three seconds or more, and these times have been commonly accepted as effective (Mesa, 2010; Tobin, 1987).

In Tobin's (1987) work on the effects of teacher wait time on discourse in mathematics and language arts classes, it was established that increasing wait time from three to five seconds produced positive changes in discourse. Mesa (2010) also found that instructors who utilized wait times greater than three seconds saw high student participation rates. Duell (1994) found that extending wait time from one to three seconds to between three and six seconds, decreased students' achievement on higher level subtests. This draws attention to Henningsen and Smith's (1997) notion that too little or too much wait time is capable of ruining activities involving higher level questioning. One may conclude that there is a cut-off time of sorts when wait time becomes less effective. The cut-off is a somewhat gray area because it has been mentioned that mathematics instructors should wait at least 10 seconds before calling on a student to respond (von Renesse & Ecke, 2014). Researchers have recommended that instructors aim for an average wait time within the three to five seconds range (Honea, 1982; McComas & Abraham, 2004; Rowe, 1986; Tobin, 1987).

In essence, instructors need to provide wait time that is sufficient for the type of question they ask (Gooding & Metz, 2013). Average wait times should be above three seconds if instructors are 1) asking students the types of questions that support their understanding of content, 2) anticipating that students are capable of producing responses to their questions, and 3) providing students ample time to generate responses. Wait times below this suggest that instructors may be asking questions they do not expect to require much thought on the students' part. On the other hand, they could be asking questions that entail much student thought but they answer the questions themselves or move on before giving students a chance to respond (Honea, 1982; Weiss, Pasley, Smith, Banilower, & Heck, 2003). Instructors have displayed difficulty maintaining wait times above three seconds, but if they can manage to do so the gains have shown to be well worth it.

The Value of Waiting

Students' opportunities to engage in their learning are shut down when diminutive wait time is offered because they are not encouraged to share their responses (Mesa, 2010). There is a push in mathematics classes to utilize discourse that promotes understanding and students' verbalization of it, but limiting the time that students have to think does not align with such results. It is necessary that instructors are cognizant of the benefits associated with providing ample wait time which go beyond giving students time to think. Numerous student incentives and changes in instructors' questioning practice are displayed when wait times greater than three seconds are implemented (Kwit, 2012; Rowe, 1986; Tobin, 1987).

Rowe (1986) provided the following 10 effects of wait time on students based on a synthesis of early wait time research:

1. The length of student responses increases between 300% and 700%, in some cases more, depending on the study.
2. More inferences are supported by evidence and logical argument.
3. The incidence of speculative thinking increases.
4. The number of questions asked by students increases, and the number of experiments they propose increases.

5. Student-student exchanges increase; teacher-centered “show and tell” behavior decreases.

6. Failures to respond decrease.

7. Disciplinary moves decrease.

8. The variety of students participating voluntarily in discussions increases. Also, the number of unsolicited, but appropriate, contributions by students increases.

9. Student confidence, as reflected in fewer inflected responses increases.

10. Achievement improves on written measures where the items are cognitively complex (p. 44-45).

Similar benefits of wait time were reported by Tobin (1987) which included that students’ achievement improved in mathematics and language arts with increased use of wait time; as measured by a mathematics test and a whole group discussion in language arts. Kwit (2012) added that when wait time is used students are able to comfortably express their thoughts aloud. However, all students will not witness such gains if instructors always call on a student by name before providing wait time (Duell, 1994). If the instructor says, for example, “Linda, how did you solve number 5?” Everyone other than Linda is liable to not give any thought to the instructor’s question because it was directed to Linda. When selecting students to answer a question it should be done after wait time has been provided for the class. Consider this, “What does it mean to be differentiable? (Pause) Nathan.”

Wait time impacts instructors in ways that have proven to reduce their execution of traditional instructional practices (Kwit, 2012; Rowe, 1986; Tobin; 1987). First, the occurrence and types of questions instructors pose change. Instructors ask fewer questions when wait times of three or more seconds are maintained, yet the amount of higher level questions increases. Second, there is a decline in instructor talk with less temptation to answer their own questions. Further, instructors display greater flexibility in facilitating discourse that is not limited to a preexisting plan. Last but not least, Rowe (1986) reported that instructors' expectations for certain groups of students increased. Certain groups included minority students and students who were not performing well.

In sum, wait time is a valuable construct that may be applied differently with different questions, and during different classroom activities. There is not a magic formula for wait time, yet pauses of three seconds or more are generally characterized as effective (Honea, 1982; McComas & Abraham, 2004; Rowe, 1986; Tobin, 1987). Teachers need to be aware of how much wait time they are using and when they are using it such that students are given fair opportunities to engage in their learning and develop needed skills (i.e., critical thinking and communication). The benefits of wait time have been echoed in literature (Gooding & Metz, ; Heinze & Erhard, 2006; Mesa 2010; Yourstone, Kray, & Albaum, 2008), but it "is just another technique if one does not understand why fostering more productive exchanges among us all is so important" (Rowe, 1986, p. 48).

Classroom Setting During Mathematics Instruction

The classroom setting during mathematics instruction can be described by the nature of the in-class activities. The in-class activities instructors use are of importance because they represent the setting(s) in which instructors pose questions. Broadly, in-class activities have been divided into two groups; those that are teacher-centered or traditional and those that are student-centered or non-traditional (Cheung, 2001; Mesa, Celis, Lande, 2013). The instructor is the transmitter of information with teacher-centered activities and students are often viewed as passive learners. Student-centered activities are different in that the instructor is not the sole owner of knowledge and students are actively engaged in learning mathematics. Contrasting views exist on which type of activities should be utilized in mathematics classrooms, yet it known that both teacher-centered and student-centered activities impact students' engagement and their satisfaction with mathematics courses (Ellis et al., 2013).

Mathematics teaching in the United States has been characterized as teacher centered and process driven according to various Trends in International Mathematics and Science Studies (http://nces.ed.gov/timss/). Lecture has been identified as the most popular teacher-centered activity, and it is often the most prevalent in-class activity used in mathematics classrooms (Blair et al., 2013; Lutzer, Rodi, Kirkman, & Maxwell, 2007). Lecture is widely defined as an oral presentation or talk directed toward students (Bergsten, 2007). The College Board of Mathematical Sciences' Fall 2010 report indicated that lecture was used in all of the surveyed courses at two-year colleges (Blair et al., 2013). Further, many courses were identified as mostly lecture (e.g., between 66-85% of the calculus level sections were majority lecture).

Lecture-discussion is one of many variations of lecture. It engages students more than the traditional lecture by incorporating questioning (O'Bannon, 2001). Even still, it has been considered a teacher-centered approach because the instructor maintains a, "I pitch, you catch" (Terenzini, Cabrera, Colbeck, Parente, & Bjorklund, 2001, p. 124) approach to instruction (O'Bannon, 2001). Fukawa-Connelly (2012) pointed out that traditional teaching is not well defined at the undergraduate level and could in fact entail instructional practices common to student-centered teaching. The researcher found that an instructor who claimed to be a traditional lecturer was more student-centered than anticipated due to the types of questions posed and the student engagement produced. This suggests that the discussion portion of lecture-discussion could in fact be student-centered and supports the notion that teacher-centered in-class activities can co-exist with those that are more student-centered; giving students exposure to a range of activities (Cooper & Robinson, 1998). We may, however, see differences in the types of questions posed in the different in-class activities.

A major aspect of reform in K-16 has been a push for instructors to move away from traditional practices of teaching mathematics to focus on those that are student centered (Bookman & Friedman, 1998; Johnson, 2013; Rasmussen, Kwon, Allen, Marrongelle, & Burtch, 2006). Student-centered approaches to teaching and learning minimize lecture in favor of students working together to solve problems while constructing their own knowledge (NCTM, 1989; 2000; Prince, 2004; Yoshinobu & Jones, 2013). Small group work, whole group discussion, and student presentations are central in-class activities of student-centered approaches (Laursen, 2013). Both small group work and student presentations allow for student to student interactions which include them explaining their thoughts. Whole group discussion also focuses on interactions, but they are generally between the instructor and students. The instructor and the students come together as a class to discuss a problem or topic during whole group discussions. It is believed that use of such activities produce students who not only conceptually understand mathematics but are capable of communicating such understanding (NCTM, 1989; 2000).

Conceptual Framework

Based on the literature reviewed above, the conceptual framework for this study was developed. A diagram of the conceptual framework can be seen in Figure 1.

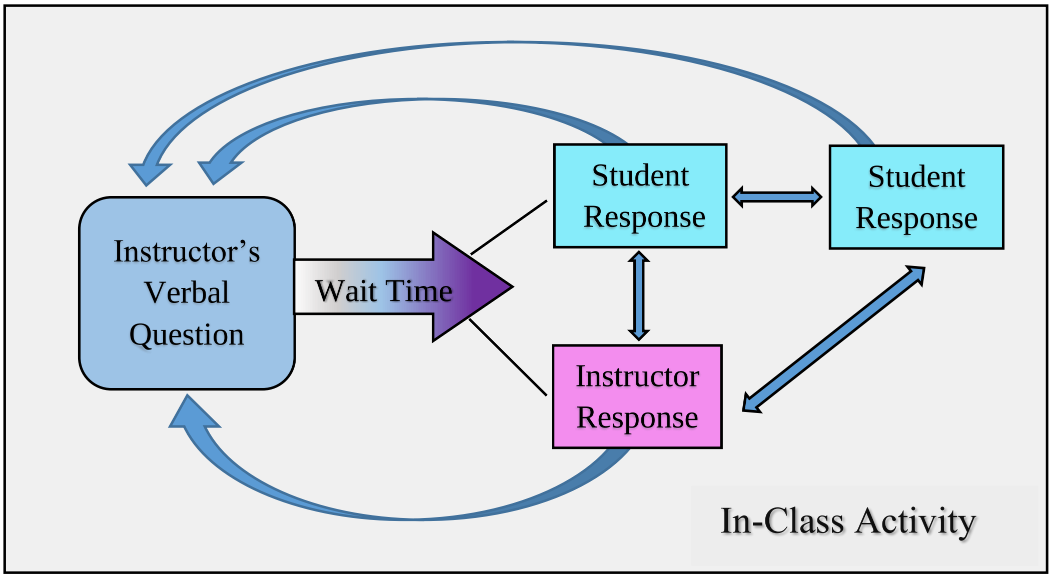

Figure 1: Conceptual framework diagram

The focus of this study, instructor's verbal questions, is located on the left and the relationship between questions, wait time, and responses is indicated by the arrows. Research has shown that these questions are capable of providing in-the-moment value for instructors and students, opening the door for discourse, and facilitating it once it has entered (Martino & Maher, 1999; Roach et al., 2010). As noted by researchers wait time is an important aspect of teacher questioning as it can positively impact both the students and the instructor if allocated appropriately (Kwit, 2012; Mesa, 2010; Rowe, 1986; Tobin, 1987). It has been established that instructors should aim for average wait times of three to five seconds (Honea, 1982; McComas & Abraham, 2004; Rowe, 1986; Tobin, 1987), however, higher wait times have been recommended (von Renesse & Ecke, 2014). In order to capture the fact that instructors provide varying wait time after asking questions, gradients were placed within the wait time arrow of the conceptual framework.

The initial utterance after wait time is either a student or instructor response, as indicated by the two nodes off the wait time arrow. It is understood that responses are not limited to answers and could be questions, explanations, or other responses. Two-headed arrows work with curved arrows to represent the interaction patterns that may take place between the instructor and the students. Multiple interaction patterns are possible; each beginning with the instructor's verbal question followed by wait time or the lack thereof. Interaction patterns are not a focus of this study, however I did attend to who responded to understand how the instructors' questions were used.

There are a number of in-class activities that may take place in an undergraduate mathematics class (e.g., lecture, small group work, whole group discussion). Instructors' mathematical questions are embedded in these in-class activities, therefore they contribute to understanding how their questions are used. This is shown in the visual of the conceptual framework by the rectangle that encompasses the flow of discourse based on an instructor's verbal questions. In short, the conceptual framework shows that instructors pose mathematical questions during in-class activities that are either responded to by a student or the instructor themselves after a period of wait time resulting in a range of potential interactions between the instructor and the students.

Chapter Summary

This chapter provided a brief overview of the value of questioning before taking a detailed look at how instructors' questions have been characterized in literature. In particular, questions have been characterized by levels, patterns, and types. Literature on wait time was reviewed because of its importance in implementing questioning. The overall point to be taken away is that mathematics instructors ask a variety of questions during class time that give students different opportunities to engage, and they do so within in-class activities. Questions should be followed by wait time so that students are given time to generate responses, yet this is not always the case.

CHAPTER 3: RESEARCH METHODS

The intent of this qualitative collective case study is to illustrate the ways undergraduate Calculus I instructors are using their mathematical questions during class. This study attends to the gap in research on teacher practices in undergraduate mathematics education research (Speer, Smith, & Horvath, 2010) by giving specific attention to questioning in Calculus I classes. Documenting this phenomenon extends what is known about the types of questions instructors pose, the way(s) instructors utilize wait-time, and the classroom setting(s) where questions are posed. Literature on teacher questioning (e.g., Boaler & Brodie, 2004; Mesa, 2010; Mills, 2013; Rowe, 1986; Shahrill, 2013; Stolk, 2013) provided a valuable lens for defining and analyzing the questioning of five undergraduate Calculus I instructors.

This chapter details the methods that were employed in answering the research questions. As previously stated, the research questions are:

How are mathematics instructors using their mathematical questions in undergraduate Calculus I classes?

Specifically:

a. What types of questions are asked by Calculus I instructors?

b. In what ways do instructors utilize wait time with their mathematical questions?

c. In what class setting(s) are the various question types and wait time observed?

The chapter begins with a general explanation for using a qualitative research design in the study. It is followed by the rationale for using the case study approach which includes attention to why I chose collective case study. The conceptual framework, data collection, and data analysis are discussed before providing a summary of the chapter.

Research Design

A qualitative research design was applied during this study because of the affordances offered by the approach. Qualitative research focuses on "naturally occurring, ordinary events in natural settings, so that we have a strong handle on what 'real life' is like" (Miles et al., 2014, p. 11). As such, the researcher acts as an instrument in the field that collects purposeful data but does not attempt to control or manipulate the phenomena being explored (Creswell, 2013; Patton, 2002). This type of research is an inductive practice that grants the researcher with ownership of discovering that which should be extracted from the data (Creswell, 2013; Frattaroli, 2012; Patton, 2002). Further, qualitative inquiry pays ongoing attention to process and change (Frattaroli, 2012; Patton, 2002). This is important for the study at hand because an instructor's questioning in an undergraduate calculus class cannot be described or understood if it is viewed as static. Qualitative research is also associated with rich descriptions (Creswell, 2013; Patton, 2002) that allow researchers to vividly illustrate their results. There are numerous qualitative approaches, but the collective case study approach was selected for answering the research questions in this study.

Rationale for Case Study Approach

Why case study? Creswell (2013) describes case study research as:

> a qualitative approach in which the investigator explores a real-life, contemporary bounded system (a case) or multiple bounded systems (cases) over time, through

> detailed, in-depth data collection involving multiple sources of information (e.g. observations, interviews, audiovisual material, and documents and reports), and reports a case description and case themes (p. 97).

An instructor's questioning is a real-life aspect of their teaching practice that cannot be fully described or understood if it separated from the classroom setting, making case study research appropriate for its exploration. Specifically, a collective case study design was selected because it is an instrumental study extended to several cases (Stake, 1995). Instrumental case studies place the case second to understanding something else making it a valuable means to gain understanding of this specific teaching practice without focusing on the instructor(s) themselves (Baxter & Jack, 2008; Johnson & Christensen, 2008; Stake, 1995). Using a collective case study approach adds confidence and robustness to findings because it allows researchers to analyze data within and across multiple cases opposed to solely isolating a single unique case (Baxter & Jack, 2008; Miles et al., 2014).

The discourse in five undergraduate Calculus I classes was studied to provide insight into the ways instructors are using their mathematical questions in Calculus I. The verbal communication that takes place during a Calculus I class was needed to understand how instructors are using their mathematical questions, therefore, the case and the unit of analysis were defined as the mathematical discourse in the individual instructors' Calculus I class. The individual cases are bounded by time and location. In this situation, each case is specifically bounded by the Spring Semester 2015 and five institutions of higher learning in the southeastern region of the United States.

Sites and Participants

Thirteen institutions including universities, colleges, and community colleges in the southeastern region of the United States were identified as potential sites for identifying participants in this study. I selected all potential sites based on proximity and the fact that they offered Calculus I during the spring 2015 semester. The final site selection was dependent on participant selection because it was not the purpose of this study to explore instructors' questioning at specific institutions.

The chair of the appropriate department (e.g. mathematics, mathematics and computer studies) at each institution was contacted, via email, asking them to identify members of their department who would be teaching Calculus I during spring 2015 and may be interested in participating in a dissertation study. The email can be found in Appendix A. Four of thirteen department chairs responded which led to alternative routes of identifying potential participants. Other points of contact were made at various institutions seeking the same information. Due to a low response rate, I utilized online course schedules to identify instructors who would be teaching Calculus I and contacted them individually.

Initial contact with potential participants was in the form of an email that included the broad goals of the study and what I was asking of the instructor. The email can be found in Appendix B. The instructors were asked to participate in a semi-structured interview. They were informed that the interview would be used for participant selection, and that approximately six hours of observation would be required for the instructors selected for the study. The interview protocol can be found in Appendix C. The interview protocol was developed with two of my committee members, and it posed questions that aimed to inform me on things like the skills the instructors wanted their Calculus I students to have upon leaving their class and what a typical class session looks like.

It was anticipated that five participants would be selected out of at least 10 potential participants, however, exactly six instructors responded and volunteered to be interviewed. One of the instructors taught high school students in a community college setting and was excluded because of my focus on undergraduate classes. The remaining five instructors were selected as participants for this collective case study. Each instructor came from a different institution; three universities, one college, and a community college.

Consent forms (in Appendix D) were mailed to the instructors after they had been informed of their selection as study participants. I addressed all questions that the instructors had, scheduled classroom observations, and proceeded to data collection. The participants will be referred to as Instructor D, Instructor P, Instructor F, Instructor T, and Instructor C. They are introduced below in the order that I began collecting data in their Calculus I class using information gathered from the semi-structured interview.

Instructor D. Instructor D was in her seventh year as a mathematics lecturer at a Research I university at the time of the study. Her experience with teaching has been strictly at the collegiate level. Before joining the faculty she was on at the time, she taught full-time at another research university. She pointed out that college teaching has been what she envisioned, but she “does not know much about teaching in general.” Instructor D has worked as a statistical researcher, but her desire to interact with people led her into the classroom.

Instructor D described her typical Calculus I class session as primarily lecture with her talking in the front. Students are provided notes in advance, and “they are supposed to be ready.” The class was identified as a large lecture that does not entail much group work. There are pauses for students to work on problems before going over it as a class, and sometimes they are given opportunities to work with the person beside them. The students are generally taking notes and asking questions during class.

It is usually the next day when Instructor D knows whether or not her students have learned what she was hoping for. This is because students have then begun to do their homework and they start to ask homework questions. When asked what are some important skills she would like her students to have upon leaving her class Instructor D stated, “This class is to help them build for the next, so you want to make sure they have all the skills like derivatives.” She also wants her students to have a good idea of how to study mathematics since many Calculus I students have always “gotten math…”

Instructor P. At the time of the study, Instructor P was in his final semester as a Professor of Mathematics at a small private liberal arts college. He had been teaching at the same college for more than forty years and pointed out that his experiences there as a student were part of the reason he wanted to teach mathematics. There was also a strong family influence. Instructor P is one of five educators in his generation, and among them lies more than 180 years of experience in the field. Teaching has been what he anticipated; however, he does not feel anyone can “envision what the whole thing is ahead of time…”

The first thing Instructor P does when he goes into every class is ask the students if they have any problems that gave them difficulty or questions. He then “moves forward from there" meaning he never really knows ahead of time what will happen during the class itself. Instructor P always has something to continue with after the students' questions are addressed, but has found that it is impossible to write down day to day lesson plans for his class because he does not know where they will be. This is primarily because his class engages in an inquiry based learning type of activities. The objective is for each class to be "conversational" with as much student participation as possible. During the typical class session, Instructor P wants to "ask questions, get their responses, and try to ask the next question so that we lead to something that yields understanding."

Instructor P does not reflect at the end of class about whether or not his students have learned what he was hoping for because most of them will have talked to him during class. The students' answers to his questions allow him see how they are thinking, but when they do not respond he does not know if they understood until the concepts show up again. The number one skill that Instructor P would like students to have upon leaving his class is "the ability to think through things and understand that there is a lot that underlies the mathematical manipulations." He wants students to leave knowing how to express their thoughts in English and mathematical language; therefore, they are required to show their work and explain in words what they did and why on graded assignments. Instructor P also feels that students should be able to identify when a problem involves differentiation and when it involves integration.

Instructor F. Instructor F, an assistant professor, was in her fifth year at a liberal arts and research university at the time of the study. She began her studies with the idea that she wanted to be a mathematician, but explained that perhaps she "did not know at that level what that meant." She did several post-docs with research and teaching but felt that teaching would be a part of her career. Instructor F envisioned teaching as helping other people understand mathematics.

The typical Calculus I class for Instructor F begins with a five minute presentation led by an individual student that reviews the previous lesson and/or discusses a homework problem. This presentation is then evaluated by the remainder of the class. Instructor F comments on the presentation, corrects notation where appropriate, and takes questions from the students before giving them a quiz. This part of the class was identified as "not very standard" compared to the remainder of the class which is lecture. Instructor F has her students "guess" what theorems will say as she writes examples to "kind of guide their intuition about what the theorem may be."

Reflection is not a common practice after class in terms of identifying if students have learned what was hoped for. Instructor F did, however, point out that when many of the students are "engaging easily and rapidly" that she felt they were on top of things. She wants her students to leave her class knowing how to use their textbook as a resource because she sees the lack of being able to do so as part of the reason students are failing calculus. Further, Instructor F wants her students to be able to keep track of their understanding so that they are able to fill in the necessary gaps. It is also important that students leave as critical thinkers who ask why and pose questions about what they do not understand.

Instructor T. Instructor T was a doctoral candidate, in mathematics education, working his first year as a full time mathematics instructor at a private liberal arts university at the time of the study. He has always wanted to teach mathematics at the undergraduate level and has taken a path through teaching at different levels to get him to that point. Instructor T has taught middle school, high school, and community college, and has served as an adjunct instructor at a university. Teaching mathematics has been personally satisfying for Instructor T and "in some regards" it has been what he envisioned it would be.

When asked to describe his typical Calculus I class-session, Instructor T explained that his class starts with a warm-up that either revisits a topic that the students have previously been introduced to or gets "their mental wheels turning about what is coming up." During the warm-up the students are encouraged to discuss their work with a peer, but it is addressed as a whole class. The class moves into a challenge problem or set of problems after the warm-up where the students are again working and communicating with each other. A whole group discussion with minimal notes occurs before wrapping up class; however, if time permits they move on to something else. Mathematica, a computational software program, is also a common element of Instructor T's Calculus I class sessions.

Instructor T sometimes reflects on whole group discussions, but he normally knows whether or not his students have learned what he was hoping for because of his use of "formative assessment." He also walks around and listens to students' discussions with their peers as they work on both the warm-up and challenge problem(s). Some important skills that Instructor T would like his students to have upon leaving his class include knowing how to communicate their ideas and what it means to give a conceptually based answer and go beyond procedure. Students should also have the basic skills of Mathematica mastered and know how to use the tool appropriately.

Instructor C. Instructor C was in his fifth year as an instructor at a community college. There was a time that he considered himself not so good in mathematics, but a teacher in his senior year of high school encouraged him to make mathematics a concentration in college. It was during college that he realized he was good at mathematics and it would be the subject he would teach. Instructor C taught high school in the Philippines and the United States prior to his current position. When asked if teaching was what he envisioned, he said it is “the only thing I know.”

The typical Calculus I class session for Instructor C entails approximately 30 minutes of lecture before the students practice everything they have covered that day. A worksheet serves as practice for the students before they come to the board and “talk about how they got to the correct answer.” Everyone shares in any given class session, and the instructor directs students’ questions about problems to the person presenting at the board. The students do not work in groups because there are only seven of them, but they are allowed to discuss with each other before going to the board. There is a lot of practice and sharing.

Instructor C gauges if his students have learned what he was hoping for based on the students’ work on the board and the discussions that transpire. He also “observes how they answer homework” and uses test results as a means of gauging students’ understanding. When students leave Instructor C’s class he wants them to be able to say they have learned something. Grasping the basic concepts is important, but students must also understand the real application of concepts.

Students. In order to capture both the questions posed by instructors and the students’ responses, all students enrolled in an instructor’s calculus course were asked to participate in the study. The study required nothing of the students other than them learning in their natural environment, therefore, their consent form informed them of the study goals and requested their agreement to be video-recorded during class sessions. Students' written consent was obtained by their respective instructor and returned to me. The consent form can be found in Appendix D. In compliance with the corresponding institutional review board (IRB), the students' identity was protected and they were not harmed in any way.

Data Collection

Classroom observations served as the source of data for this study because the classroom is where instructors' questioning takes place. Additionally, observations are highlighted as one of the most common methods of data collection for the case study approach (Creswell, 2013; Patton, 2002; Yin, 2009). Each instructor's Calculus I class was observed for approximately 6 consecutive hours of instruction during spring 2015. All observations occurred at the place and time that the course was normally taught such that the instructor's questions and the students' responses were captured in their natural environment. The length of each observation was contingent upon the scheduled class time; they lasted between 50 and 140 minutes. The observation time per instructor can be seen in Table 2.

Table 2: Overview of Observation Times

Instructor	**Class Time**	**Number of Times Observed**	**Total Observation Time**
D	12:25-1:15 pm	7	5 hrs. 50 min.
P	8:30-9:45 am	5	6 hrs. 15 min.
F	2:00-3:15 pm	5	6 hrs. 15 min.
T	12:15-1:25 pm	5	5 hrs. 50 min.
C	6:00-8:20 pm (20 min break included)	3	6 hrs.

I decided to take on the role of a non-participant observer, as to not impact what was being said and done in the classroom. Further, being a non-participant observer meant there was more time to focus on what was going on in the classroom (Creswell, 2013). Field notes were taken using an adapted observation protocol from Creswell (2013) to increase sources of evidence (Yin, 2009). This allowed me to document what I was seeing and thinking when observing each class. The observation protocol can be found in Appendix E. The observation protocols were edited for typos after each classroom observation and saved on a personal password secured laptop and a personal external hard-drive.

The observations were also video- and audio-recorded to assure that as much verbal dialogue as possible was captured. Each instructor wore a lapel microphone attached to a video recorder that followed them around the classroom to pick up what was said as they moved around. Two additional video-cameras were used; one focused on what was being shared and/or written on the board and the other captured the students. Students who did not agree to be video-recorded were placed out of the camera's line of sight. The student view was only recorded one day for Instructor F due to the discomfort of some of her students. All observations were audio-recorded for the purpose of capturing as much student talk as possible. Audio recorders were placed in various spaces around the classroom. At most four audio recorders were utilized based on the size of the classroom and the number of students in the class. After each observation, data was uploaded from audio- and video-recording devices to a personal password secured laptop and a personal external hard-drive that is stored in a locked location.

Data Analysis

Qualitative analysis is described as transforming data into findings (Patton, 2002). This transformation can be viewed as a spiral process that entails preparing and organizing, reading and memoing, coding, interpreting, and representing the data (Creswell, 2013). For this study, the data was organized into folders per instructor during data collection. I transcribed the classroom observation data verbatim at the conclusion of the data collection phase. The main video-recordings were used for transcribing, and the audio-recordings served as cross-references when necessary. The main video-recordings were also uploaded into Atlas.ti 7.0 and further used for calculating wait time. The transcripts were uploaded into Atlas.ti 7.0 and used for coding.

Identifying questions. I read through each transcript numerous times and highlighted all cases of the instructors' questions giving particular attention to mathematical questions. Mathematical questions are utterances that seek a response from students in regard to mathematics. As such, utterances that take the form of questions (e.g., “Can you explain where 37.8 came from?” and “What’s the area of a circle?”) as well statements that function as questions (e.g., “Ok someone tell me the antiderivative of $32t$.” and “Explain how you got that.”) were analyzed in this study. This aligns with Boaler and Brodie’s (2004) work; as does the inclusion of questions not directly related to the mathematics content that were used for contextual purposes (e.g., “Have y’all ever seen Jeopardy?”). Questions that asked students the problems they wanted to do or what they wanted to go over in class were also counted in the analysis because they suggest the mathematics to be discussed. All other questions not related to mathematics were omitted (e.g., “Can I write something on your paper?” and “What’s the date?”).

The most difficult decision faced in terms of identifying mathematical questions involved the inclusion of statements followed by questions like “Ok?” or “Right?” Initially I intended to include “Ok?” as a question but found that I could not say with certainty when it was a question. This would involve a fine-tuned knowledge of intonation found in discourse analysis (Brown & Yule, 1983) which is beyond the scope of this study. For example, Instructor F said “This example shows us that the other way is not possible. It is not necessarily true, so just because you have a critical point does not mean you have local minimum or maximum.” She followed this with “ok” before going on to say “So that was what we discussed a week ago…” It was difficult for me to determine if “Ok?” followed the first sentence as a question or if it was simply an “Ok” at the start of the subsequent statement. These utterances were rarely followed by a student response, so I decided to omit statements followed by “Ok?” from the analysis. Statements followed by “Right?”, “Correct?”, “Isn’t it?” and similar questions were initially going to be included for analysis under the condition that they were verbally answered. It was later established that there were instances that the instructors posed these types of questions and provided wait time without getting a verbal student response suggesting that these instances should also be considered for analysis.

Microsoft Word was used to create tables for everything that had been established as a mathematical question for each classroom observation. The line number(s) from the transcripts for each question were documented, the questions were listed, and the start and stop times needed for coding wait time were recorded. It was during the production of tables that questions were omitted and/or added if they were previously missed.

Calculating wait time. Wait time was calculated using the classroom observation videos that were uploaded into Atlas.ti 7.0 and a wireless mouse. Atlas.ti 7.0 allowed time stamps to be captured to the thousandths place. Wait time is identified as the gap of time between the final utterance in an instructor’s question and the initial utterance of the person who speaks afterward. Time stamps for these utterances were captured for each question and used for coding wait time.

Wait times were calculated three times for more than 100 questions to establish a sense of reaction time and accuracy. It was observed that for any given question, the calculated wait times had a maximum range of 0.2 seconds. I used this information to support calculating the wait time of all other questions one time under the assumption that it could be approximately .2 seconds off. Table 3 provides an example of questions where wait time was calculated three times per question. For all other questions time stamps were documented once and wait time was mentally calculated during coding.

Table 3: Examples of Wait Times Calculated Multiple Times

Line #	Question	Wait-Time
1) 113	The concavity changes at x equals two, and how do we know?	17:10.276-17:11.505; (1.229) 17:10.354-17:11.411; (1.057) 17:10.229-17:11.460 (1.231)
2) 127	What does it tell us about the graph of f and inflection points?	18:52.700-18:53.867; (1.167) 18:52.639-18:53.800; (1.161) 18:52.755-18:53.831 (1.076)
3) 127	What do we get for 7c?	18:54.844-19:01.293; (6.449) 18:54.868-19:01.169; (6.301) 18:54.809-19:01.228 (6.419)

Coding. All coding was done in Atlas.ti 7.0 and began after the time stamps needed for calculating wait time were recorded for all questions. Each question was coded using codes adapted from literature and guided by the conceptual framework found in Figure 1 on page 35. The questions were assigned a code related to type, wait time, response, and in-class activity. The abovementioned categories of codes were used simultaneously to code each question, meaning a question was assigned a code for each before moving to the next question. The data was coded in the order in which data collection began; Instructor D, Instructor P, Instructor F, Instructor T, and Instructor C.

The question type was assigned according to a priori codes from Boaler and Brodie's (2004) categories of questions discussed in the literature review. Boaler and Brodie (2004) identified nine types of questions which set the base for the question types used in this study. An additional question type, *Other,* was created to capture questions that did not fall into Boaler and Brodie's (2004) questions types but were mathematical nonetheless. The questions were coded for type based on chunks, indicating that the talk that occurred around the instructors' questions was considered. They were recognized as one of 10 types and coded as such during an inter-relater reliability exercise between a colleague and myself. We worked to achieve clarity on the codes and achieved 90% reliability.

Two additional question types emerged during coding that were not in Boaler and Brodie's (2004) work. The instructors asked questions to *Request Validity* and *Check Understanding* both of which aligned with question types described in literature (Mesa, 2010; Mills, 2013; Stolk, 2013). This included questions like "Is that right?" and "Does that make sense?" respectively, that were initially coded as *Other*. The *Other* category was exhausted once its questions were assigned to the emergent codes. Each code is described with exemplars from this study's data in Table 4. This information was used to answer research question 1a.

Table 4: Adapted Boaler and Brodie (2004) Categories of Questions

Question Type	Description	Examples
1. Gathering Information	Requires immediate answer Rehearses known facts or procedures Enables students to state facts, procedures, or next steps in them	C: "What's the derivative of secant?" P: "Anyone know what the area formula is for a trapezoid?" F: "What did we talk about last time?"
2. Inserting Terminology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them	P: "Anybody know what that's called?"
3. Exploring mathematical meanings and/or relationships (Meaning/Relationships)	Points to underlying mathematical relationships and meanings. Makes links between mathematical ideas and representations	T: "Can someone tie in this negative slope to the shape of this here?" C: "So where is the five miles in the right angle?"
4. Probing	Asks student to articulate, elaborate, or clarify ideas	P: "Say how you got that [Student]." T: "Can you give me an explanation?"
5. Generating Discussion	Solicits contributions from other members of class.	T: "[Student] anything to add or question?" P: "Particular problems?"
6. Linking and Applying	Points to relationships among mathematical ideas and mathematics and other areas of study/life Allows students to provide examples of mathematical ideas	F: "So what's the difference between a local minimum and a global minimum?" F: "Can you give me an example?"
7. Extending Thinking	Extends the situation under discussion to other situations where similar ideas may be used.	D: "What if it had been the limit as x goes toward plus infinity and it had been x squared e to the x?"
8. Orienting and Focusing	Helps students to focus on key elements or aspects of the situation in order to enable problem-solving.	F: "What am I trying to do?"
9. Establishing Context	Talks about issues outside of math in order to enable links to be made with mathematics.	D: "Have y'all ever seen Jeopardy?" P: "Anybody ever have this happen to them?"
10. Requesting Validity	Asks students to consider the validity of a claim.	C: "Ok that's the u. Correct?" T: "That'll be like 7.5. Right?"
11. Checking Understanding	Asks students to reflect on their understanding of explanations, problems or concepts.	D: "Does that make sense?" P: "Questions about this stuff?"

Wait time was coded using codes adapted from literature. Mesa (2010) included *Immediate Student Response, Questions No-Wait,* and *Questions Wait* among their categories of questions. *Immediate Student Response* was assigned to questions that immediately received a student response, *Questions No-Wait* was reserved for questions that the instructor waited two seconds or less for a response, and *Questions Wait* was assigned to questions that the instructor waited three seconds or more until a response was given. In this study, *Immediate Student Response* was used in place of *Question Answered* such that it is not assumed that the response had to be a student answer. I utilized the researcher's description of *Questions No-Wait* to further detail *Immediate Student Response* as a question that received a student response in two seconds or less. *No Wait-Time* was used synonymously with Mesa's (2010) *Questions No-Wait.* I included *Minimal Wait-Time* to capture pauses between two and three seconds exclusive. *Wait-Time* represented a pause of 3 to 5 seconds prior to a response. *Extended Wait-Time* was defined as a pause of more than 5 seconds before a response. The responses to the instructors' questions took several forms and resulted in emergent codes, namely, *student answer, student question, student comment, instructor answer, instructor question, instructor comment, instructor explanation,* and *acknowledgment.* Examples of each response code from the data can be found in Table 5. This information was used to answer research question 1b.

Table 5: Response Code Exemplars

Response Code	Example
Student answer	F: "Where's it increasing?" S: "Negative pi over two to pi over two."
Student question	T: "Rickie anything else?" S: "S: I was just going to ask like you said the two x, if it was two x then the slope, then it would have been two x?"
Student comment	D: "What function has derivative e to the x?" S: "Say the question slower."
Instructor answer	C: "So the derivative, the derivative of x squared plus y squared is equal to 25 is what? (Pause) I have to write that, then this is 2x, I need this one is not constant so x prime plus y is being asked so 2y, y prime is equal to zero.
Instructor question	F: "so what am I going to see about the derivative to the left of this x equal c line? Is this derivative positive or negative?"
Instructor comment	P: "Linda is this alright with you? (Pause) We're finishing up the problem we started on Monday. You weren't here to see it."
Instructor explanation	D: "Does that make sense? (Pause) And then now when you plug in zero or you evaluate the limit as x approaches zero you're getting cosine of zero which is one and then a one up top or you're getting minus one for that limit."
Acknowledgment	T: "What insight did you or people at your table have on tackling this antiderivative question? (Pause) Nathan I see you hand."

The codes assigned for the classroom setting were established based on the in-class activities observed during the Calculus I classes. There was evidence of *Lecture-Discussions, Individual Work, Small Group Work, Whole Group Discussions,* and *Student Presentations.* Each in-class activity was understood as mutually exclusive. For clarity purposes, both *Lecture-Discussion* and *Whole Group Discussion* are whole group activities, yet *Lecture-Discussion* involves the instructor presenting information to the students and *Whole Group Discussion* does not. Further, *Whole Group Discussion* is used when the instructor and the students come together to discuss a problem or problems. This information was used to answer research question 1c.

Case analysis. Within case and cross-case analysis were used to determine findings. Creswell (2013) pointed out that this is a routine practice for collective case studies. The within case analysis began after all of the data was coded. Each instructors' class represented a case of discourse in Calculus I where questions could be found as such the findings were compiled by instructor. I utilized Atlas.ti 7.0 to aid in organizing and analyzing the coded data and produced rich descriptions around question types, wait time, and classroom setting for each instructor. I looked across the observed class sessions to determine themes and findings. Similar to the within case analysis, my cross-case analysis findings were described according to question types, wait time, and classroom setting. The instructors were compared in these areas for the purpose of identifying themes across the instructors. My within case and cross-case finding are reported in Chapter Four.

Research Validity and Reliability

Measures were taken to assure that this study is both valid and reliable. As suggested by Miles et al. (2014), both internal and external validity were addressed when considering the quality of this qualitative research. Internal validity, also identified as credibility or authenticity, can be summed up as the "truth value" (p.312) of the study. This study attends to points provided by Miles et al. (2014) including meaningful, context rich descriptions and unified findings. Additionally, areas of uncertainty were acknowledged. Unlike internal validity, external validity gives attention to the generalizability of a study's findings (Yin, 2009). This study included five cases and provides detailed descriptions such that it can be replicated with other samples, and care has been taken to ensure that the final write-up of the study is logical yet persuasive. Each instructor was observed for six hours, resulting in repeated observations which also increases validity (Merriam, 1998).

Reliability is related to the dependability and auditability of a study (Miles et al., 2014). Attention was given to the goal of reliability which is to minimize the biases and errors in a study (Yin, 2009). Again, using a list of points to consider provided by Miles et al. (2014), all biases are expressed, peer review was in place, and analytic constructs were specified. This work was conducted with integrity, being sure to thoroughly document the processes followed during the study.

Ethical Considerations

Before conducting this study, consent was granted from the North Carolina State University IRB. I sought potential participants and conducted semi-structured interviews before sending consent forms for participant agreement. All consent forms were signed before data collection began. This study did not pose any harm to the participants, however, a major part of the consent was agreeing to be video-recorded during class sessions. In the event that the students felt uncomfortable, they were not video-recorded. The videos were not viewed by anyone other than myself during this study, yet the participants agreed to being shown in excerpts when I report results. Each instructor as well as the research sites were assigned pseudonyms. The students were generally identified by S, but pseudonyms were assigned where applicable. Audio- and video-recordings were immediately uploaded onto my personal computer and deleted from the recording devices. The video-recordings were uploaded into Windows Media Player before being uploaded in Atlas.ti 7.0. Field notes and transcripts were typed up and saved in Microsoft Word. All of the data collected was stored on a personal password protected computer. Additionally, it was backed up on a personal external hard drive which was stored in a locked location and on NC State University's Google Drive.

Subjectivity Statement

As a former high school mathematics teacher and an adjunct mathematics instructor at a liberal arts university, I openly acknowledge that I have biases. I believe that a core tenet of reforming K-16 mathematics is the effective use of discourse in teaching and learning. Instructors play a major role in producing opportunities for students to engage in their learning. This involves instructors' asking questions that engage all students in conversations about mathematics that go beyond knowing algorithms and number crunching. My beliefs on the value of instructors' questioning in teaching and learning mathematics are due to the knowledge that I have acquired in my doctoral program, which has included some intense reading of literature. My views on classroom discourse were not used to personally judge the instructors or the students in any way. I remained objective during data collection and the analysis that followed. My goal was to understand and describe how instructors are using their mathematical questions in Calculus I classes, not to implement an intervention. As a result, full attention was devoted to that which was observed.

Chapter Summary

This chapter provides an in depth description of the methodology of the proposed study. The stage is set at the start of the chapter with the presentation of research questions.

The research design is discussed in detail, including the rationale for conducting qualitative research with a collective case study paradigm. Data collection approaches as well as analysis were specified. Attention was also given to validity and reliability strategies, ethical issues, and researcher subjectivity.

The findings of this study are detailed in the following chapter. Rich descriptions of within case findings are presented prior to cross-case analysis findings. Both within and cross-case findings are organized by question types, wait time, and classroom setting.

CHAPTER 4: CASE ANALYSIS AND FINDINGS

The purpose of this chapter is to provide vivid descriptions of the ways undergraduate Calculus I instructors use their mathematical questions. This entails describing the types of questions asked, the wait time provided, as well as the classroom setting(s) (i.e., in-class activities) in which the questions were posed. This chapter is organized in six sections; one for each of the case instructors whose class was observed and one reserved for cross-case analysis. The first five sections represent within case findings found from approximately six hours of classroom observations. Each case begins with an overview of the classes. The findings about the types of questions posed, the way wait time was utilized, and the classroom setting are then presented with examples within their own subsection. The examples are taken directly from the transcripts and are provided to illustrate how questioning was used in the observed classes. At times the examples include student responses, however, the students' responses were not the focus of the study. As such, there was no attention devoted to the correctness of students' answers. The cases are organized in the order they were observed in, namely, Instructor D, Instructor P, Instructor F, Instructor T, and Instructor C.

Case of Instructor D

Instructor D's class was held in a lecture hall with tables organized in symmetric halves with six rows of various lengths. There was a fairly wide aisle in the center of room, and on either side of it you would find 45 seats. Two large dry erase boards were on the front wall. Both boards had a screen that could be lowered in front of them. A computer, projection system, and document camera sat in the front of the room at the center where Instructor D stood most of the time. The document camera was frequently used and the computer screen was sometimes displayed to show a problem.

Between 55-67 students attended class when it was required. They had an optional review day and what Instructor D called a “wild card day” where there were 20 and 11 students present respectively. The wild card day was spent discussing what students wanted to go over which, during this study, happened to be homework problems. On any given day, you could observe Instructor D standing at the front of the room lecturing and writing on paper that was displayed with a document camera. She wrote very detailed steps as she provided notes and worked through problems. The students were not required to record everything the instructor wrote because they had access to her notes prior to class.

Occasionally, students were asked to work individually or with a partner on a problem or part of a problem. These problems were generally displayed from the document camera, but in two instances the students were given a handout of problems to work on. Instructor D walked through the room checking in with students and looking at their work during these times. For the most part, students spent much of their time listening to the instructor because she did most of the talking. The instructor commonly said more than a 100 words during her talk turns. In multiple instances her talk turns included more than 600 words. In one particular case she said more than 1000 words before a student said something. These talk turns entailed the instructor explaining, commenting, asking questions, and at times answering them. Instructor D covered a variety of topics during this study and engaged students in a few in-class activities. Table 6 provides a general overview of each observed day.

Table 6: Overview of Instructor D's Class by Day

Day of Observation	Overview of Class Session	Classroom Setting
Day 1	• Reviewed L'Hôpital's Rule and four indeterminate forms ($\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \times \infty$, and $\infty - \infty$) • Introduced remaining indeterminate forms (0^0, 1^∞, and ∞^0) and went over examples	• Lecture-discussion • Option to work with a partner twice; finding the derivatives needed to use L'Hôpital's rule • Whole group discussion
Day 2 (Optional Review Day)	• L'Hôpital's Rule • Local minima/maxima • Optimization	• Lecture-discussion
Day 3 (Required Review Day)	• Logarithms & their derivatives • Linear approximations • Absolute maxima and minima • Mean Value Theorem • First and second derivative tests	• Lecture-discussion • Option to work with a partner; linear approximation of a function at a point
Day 4	• Introduced Newton's Method and went over examples • Review of basic derivatives • Introduction of antiderivatives with practice	• Lecture-discussion • Worked with a partner on Newton's Method • Whole group discussion • Individual work on worksheet to review derivatives • Whole group discussion • Individual work on worksheet to find basic antiderivatives • Whole group discussion
Day 5 (Optional Wildcard Day)	• Homework problems on Newton's Method • Homework problems on anti-differentiation	• Whole group discussion

Table 6 Continued

Day 6	• Anti-differentiation with examples • Explored area under the curve • Introduced Riemann Sums	• Lecture-discussion • Partner work on area under the curve worksheet • Whole group discussion
Day 7	• Approximating area under the curve using Riemann Sums (left, right, and midpoint sums) • Area under the curve with summation formula	• Lecture-discussion • Students worked with a partner to find a left or right Riemann sum • Whole group discussion

Questions by Type

There was evidence of all 11 question types during Instructor D's observed Calculus I sessions as seen in Table 7. *Gathering Information* questions represented a vast 49.1% of the instructor's questions, however *Generating Discussion, Checking Understanding,* and *Requesting Validity* questions also had a noticeable presence (between 10-20% each). There was minimal evidence of questions that requested deeper student thinking with *Probing* and *Meaning/Relationships* making up less than 4% of Instructor D's questions. Additional question types were of low occurrence, but they were asked nonetheless.

Table 7: Question Types Used by Instructor D

Question Types	Frequency (Percentage)
1. Gathering Information	139 (49.1%)
2. Inserting Terminology	2 (.7%)
3. Meaning/Relationships	2 (.7%)
4. Probing	7 (2.5%)
5. Generating Discussion	28 (9.9%)
6. Linking/Applying	1 (0.4%)
7. Extending Thinking	3 (1.1%)

Table 7 Continued

8. Orienting/Focusing	1 (0.4%)
9. Establishing Context	6 (2.1%)
10. Checking Understanding	54 (19.1%)
11. Requesting Validity	40 (14.1%)
Total:	283 (100.1%)

The percent of each question type asked by Instructor D per day is displayed in Figure 2. The figure shows that the instructor asked between five and seven types of questions per day. It also shows that *Gathering Information*, *Generating Discussion*, *Checking Understanding*, and *Requesting Validity* questions were present daily. Findings from these four question types are the focus of this section in addition to *Probing* because of how the questions were used. Additionally, a single subsection is devoted to discussing the remaining types.

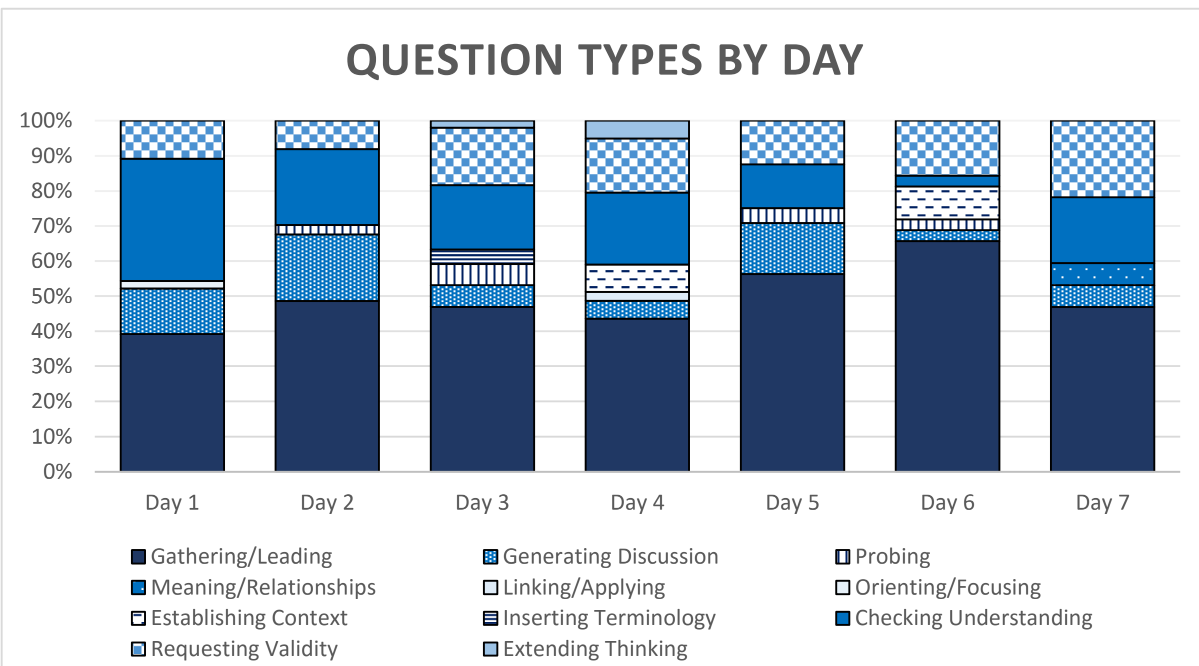

Figure 2: Instructor D’s question types per day

Gathering information. The bulk of questions identified as *Gathering Information* in Instructor D's class required the students to rehearse known facts or procedures. These questions often engaged students in completing some type of computation. The students were asked the following questions involving numeric computations in one lesson, "…So what is two minus three eighths?", "So what is three to the seventh?" and "And then what's two to the seventh…?" There was not much more evidence of arithmetic computation, yet additional questions involving computations included students finding limits, derivatives, and antiderivatives.

On each observed day there was evidence of students being asked to use known rules to compute the derivative of a function. These questions were normally sub-questions used to solve a more complicated problem during lecture. For example, students were asked to find the derivative of a function during the process of finding the minimum and/or maximum of a function. Another example involved L'Hôpital's rule which requires the use of derivatives to determine limits of functions that have indeterminate forms. One problem included the following discourse around derivatives.

I: …So what's the derivative of $\cos(mx)$?

S: $m\sin(mx)$

I: And this would be what? Plus n.

S: $\sin(nx)$

I: $\sin(nx)$, and then the derivative of x^2 would be?

S: $2x$

I: $2x$. Ok and then check form again. So now if I plug in zero. Sine of zero, zero times m is still zero. Sine of zero is zero. Zero plus zero is zero. Two times zero is zero, so I'm still zero over zero.

In this example it can be seen that the students were asked to find three derivatives, and they responded each time. This was typical of questions that asked for computations.

Other questions that *Gathered Information* from students asked them to state known facts, known processes, or next steps of those processes. The first question Instructor D asked in her observed classes was, "when are the two times, you can use L'Hôpital's rule just straight out?" This was followed by, "If you are infinity minus infinity what did we do?" She was reviewing what was previously taught when she asked these questions so the students were expected to recall information already covered in class. Additional questions of this kind were asked in other class sessions like, "So f is increasing if and only if what's true about the derivative?", "An inflection point is where the second derivative equals zero and what happens on either side of it?", and "How do you always find tangent lines?" Instructor D's use of *Gathering Information* questions suggests a focus on answers that are either correct or incorrect.

Requesting validity. During each classroom observation Instructor D asked students to validate the claims she made while explaining problems. At times, she asked the students questions that used the word "agree" like "We all agree on that?", "Y'all agree with me about L'Hôpital's rule?", and "Does everyone agree with my algebra?" Other times there were statements followed by "Right?" which also asked students to engage in confirming assertions. An example can be seen in the lecture on Riemann Sums where Instructor D introduced the idea of using summation notation to calculate the sums. After showing students how to use summation notation to find particular Riemann Sums, she pointed out that you would surely want to use this form when given a large number of sub-intervals. She said, “You wouldn’t want to have to actually add up 1000 sub-rectangles’ areas. Right?” Here she was asking students to confirm the idea that they would not want to physically add the areas of 1000 sub-rectangles indicating the usefulness of the formula. If nothing else she was giving them a moment to reflect.

There was an occasional “Is that right?” or a similar question. These questions were normally about a computation. For example, as she went through a problem using L’Hôpital’s rule she asked for the derivative of an expression. She computed the derivative herself on a piece of paper that was being projected and asked, “Is that right?” Despite the wording of these questions, students hardly ever responded verbally.

There is reason to believe the instructor was looking at nonverbal feedback. Consider the excerpt below which took place after students worked with a partner to approximate the area under a curve using a right-hand approximation. The instructor asked if anyone had an estimate for her at the start of this particular whole group discussion. A student responded, and the discussion below transpired.

> I: 13.25. Anybody else get 13.25? Yayyy! I think we’re right…And so your numbers are so the $f(.5)$ so you have $1.75 + f(1)$ is your four, $f(2)$ and then all of those multiplied together so what did y’all get? 13.25 I think. Is that right?
>
> S: Yes.
>
> I: And this one do we all agree that’s going to be an over approximation?

S: Yes.

Instructor D asked, “Anybody else get 13.25?” after repeating the student’s response to her initial question. There was no evidence of a verbal response, but she then said “Yayyy! I think we’re right” as if someone indicated that they too got 13.25. This excerpt also contains two of very few instances that students expressed their agreement aloud. They agreed with her recollection that 13.25 was the answer they provided as well as her claim that this particular sum produced an over approximation.

The students were asked to consider the validity of their classmates’ claims in a couple of instances. One example took place as Instructor D worked through antiderivative problems on one of their optional days. She explained to the students that they would have to find the antiderivative two times when they were given the second derivative of a function and asked to find the function itself. The following excerpt comes from the discussion that transpired as she led them through finding $f(x)$ given $f''(x) = sin(x)$.

I: …So what’s the anti; like go one step back gets you to f prime, so what function has derivative sine x?

S: Minus cosine.

I: Very good. Everybody agree with him? And then don’t forget to put your plus c because that will be important and then you have to anti-differentiate again…

The students did not regularly provide verbal agreement similar to when the instructor asked them to validate her claims. When students offered a verbal response, it was some form of “yes.” As previously stated, these questions were often embedded in the instructor’s explanations. This also held true for questions that requested students' examination of their own understanding.

Checking understanding. Students in Instructor D's class were challenged to examine their understanding of her explanations on a regular basis. This took place during her lecture-discussions, but also showed up as she walked around talking to students about their work and during their brief whole group discussions. Instructor D asked students, "Does that make sense?" on multiple occasions after explaining. Students did not always verbally respond, but when they did it was usually with a question. On Day 1 they were working on L'Hôpital's rule and the students were asked to find the derivative and check it with a neighbor. The following discussion transpired as she walked around looking over students' work.

> I: Um hmm, but it tells you one over tan of $2x$ and then times the secant squared. So like you did two steps at one time. So it's one over what's inside; you hold it fixed. Then you multiply by your secant squared. Does that make sense?
>
> S: Do you multiply the two as well after you do that?

It can be gathered from this excerpt that a student responding with a question does not indicate that they lack understanding of what was explained. This student's question pertained to what to do after that.

There were other questions that asked students to check their understanding of explanations that did not explicitly say, "Does this make sense?" One example occurred during Instructor D's lecture-discussion on Riemann Sums using summation form. A student asked, "What's the i mean?" The instructor explained i as:

> The counter. So i is the integers. So i is going from one to n. So i is in here and so it's like me stepping along the rectangles. It's like the first rectangle, the second sub-rectangle, the third sub-rectangle all the way up until I have n sub-rectangles.

She followed the explanation with the question, "Does that help kind of?" The student did not respond verbally to this, however the question challenged the student to analyze his understanding of the instructor's explanation. He could not provide an honest response to this question without considering his understanding of what she explained i was.

Similar questions requested that students check their understanding like, "How are you doing?" and "Any questions thus far?" She asked students about their understanding of a particular concept with, "Ok, so any questions on Newton's Method?" There was also evidence of questioning students' understanding of notation (e.g., summation notation). These questions gave students a chance to consider their understanding of explanations, concepts, or other mathematical ideas and presented potential moments for students to engage in the class discussion.

Generating discussion. From time to time Instructor D was observed asking students what they wanted to do. These questions were identified as *Generating Discussion* because they asked for contributions from the class, yet they are somewhat similar to questions that ask students to examine their understanding because when the instructor asks students what they want to do it is likely that they consider what they do or do not understand. This was observed on Day 1, for example, after the instructor finished her lecture-discussion on indeterminate forms of L'Hôpital's Rule. The instructor said, "Ok so you guys get to pick now; do you want me to do this one or do you want to ask homework questions?" A student had a question about the problem that was still being projected, so there was a small discussion before the instructor asked students again what they would like to do. Students sometimes responded with suggestions, other times they did not respond and the instructor picked problems and asked students if they were okay with them. Questions of this nature were observed on two other days, and they made up the bulk of questions identified as the *Generating Discussion* type.

There were not many occasions that the instructor asked students to share their thoughts or ideas. One day she said “So what is your theory about increasing, decreasing and derivatives?” This was considered *Generating Discussion* because she said “what is your theory” opposed to what is “the theory” indicating that she was eliciting how students thought about it. The first question Instructor D asked on Day 4 was, “So what is Newton’s Method?” This was a method that they had not discussed in Calculus I, but they may have been exposed to it somewhere else. It appeared that the instructor was asking students to share their ideas about it. It is important to note that the aforementioned questions were all followed by an instructor question. Students did not answer this type of question. They either went unanswered or were later answered by the instructor. The students did sometimes answer the subsequent question. Instructor D subtly asked for contributions from other students with the following question, “Anybody else get another approximation?” The instructor provided her own approximation after she first made a comment.

There was not much evidence of the instructor asking questions to generate discussion. Those questions that were coded as such did not produce discussion, but they still served as moments that students had to share their suggestions for what to do during class and to share their thoughts.

Probing. It was a rare occurrence (7 times in 6 hours-see Table 7) that students in Instructor D's Calculus I class were asked questions that followed-up on, or probed, their thinking. There was, however, some variation in the *Probing* questions that took place making them worthy of discussion. A couple of questions asked about a method or approach to do something. One example occurred after the students worked on a linearization problem. A student pointed out during their discussion of the problem that he knew another way to simplify. Instructor D said "Ok, go ahead, how?" Here she was asking the student to explain his method to simplifying. There was another question that probed students about an approach. It was "How do I pick which one to use?" This was in reference to deciding to use the first or second derivative test for determining if a critical point is a minimum or maximum.

A different kind of probing transpired as the instructor explained to students what she would be looking for in optimization problems on a test. She told them that the "first big part" was getting a function written in terms of one variable. This was followed with her asking the students what to do next. The selection below picks up with a student's response.

S: Take the derivative.

I: And you're taking the derivative for what purpose?

S: Set it equal to zero.

I: Setting it equal to zero, so you're finding the critical points…

After the student responds, she asked "…for what purpose?" This is equivalent to asking the student to provide the reason for taking the derivative which is like questioning why.

The other *Probing* questions observed asked the students to provide additional information. For example, Instructor D asked “Solve for r where?” after a student stated that solving for r was the next step of a problem. She requested that another student elaborate during an example on Newton’s Method after they said you would need to “set it” equal to zero. Instructor D said, “Set what equal to zero?” asking them for more information. Similar to the probing excerpt provided above, the student could have been probed further.

It was common for students to respond to this type of question. Their responses were generally followed by some form of validation or correction before the instructor took over explaining. *Probing* questions provided a few students with the opportunity to elaborate with additional information or on “how” or “why”.

Additional question types. Questions identified as *Context, Inserting Terminology, Orienting/Focusing, Linking/Applying, Extending Thinking,* and *Meaning/Relationships* each had frequencies of six or less across all observed class sessions. These questions did not have a highly visible presence, but they contribute to the range of questions Instructor D posed. One question from each of the abovementioned types can be found in Table 8. The examples provided for *Linking/Applying* and *Extending Thinking* were the only illustrations available. The students did not respond to many questions found in these additional question types. In fact, the example questions in the table were not answered by students. A few were followed by an instructor question, and others were answered by the instructor at some later point.

Table 8: Examples from Instructor D's Low Frequency Question Types

Question Type	Frequency	Example
Establishing Context	6	Have y'all ever seen Jeopardy?
Inserting Terminology	2	What are other words for that?
Orienting/Focusing	1	So this is what?
Linking/Applying	1	So when you're looking at this if you're writing a computer program what would route like an if-then jump out start again?
Extending Thinking	3	What if it had been the limit as x goes toward plus infinity and it had been x squared e to the x?
Meaning/Relationships	2	And then where did I get my heights from?

The examples for *Context* and *Inserting Terminology* were easy to identify. The other examples are explained here. The instructor *Oriented/ Focused* students' attention on something she had written and accidentally "boxed." She always boxed answers and had boxed early so she was focusing students on what that part actually was. Instructor D's *Linking/Applying* question involved linking Newton's Method to computer programming, another area of study. She utilized the *Extended Thinking* type when she asked them to consider what would happen if the problem said something slightly different. Lastly, students were given an opportunity to make *meaning* of the heights shown in an equation in regards to a graphical representation.

Summary. Despite having evidence of all 11 question types, Instructor D mainly *Gathered Information* from students as they made up 49.1% of her overall questioning. There was a lot of explaining on the instructor's part which lent itself to asking students to examine their understanding of her explanations and to validate her claims. Questions identified as *Gathering Information, Checking Understanding,* and *Requesting Validity* made up more than 80% of Instructor D's questions, however there was evidence of questions that required deeper thinking.

Wait Time

There was evidence of each wait time category observed after Instructor D's questions. The percent for each can be found in Figure 3. It can be seen that many of the instructor's questions, 38% to be exact, were coded as *No Wait-Time* indicating that the instructor frequently provided some type of response within two seconds of asking questions. Questions coded as *No Wait-Time* did not include student responses, however, the students responded to Instructor D's questions within two seconds 33% of the time as noted by *Immediate Student Response*. The students were not restricted to answering questions promptly as 29% of the instructor's questions were followed by either *Minimal Wait-Time, Wait-Time,* or *Extended Wait-Time*.

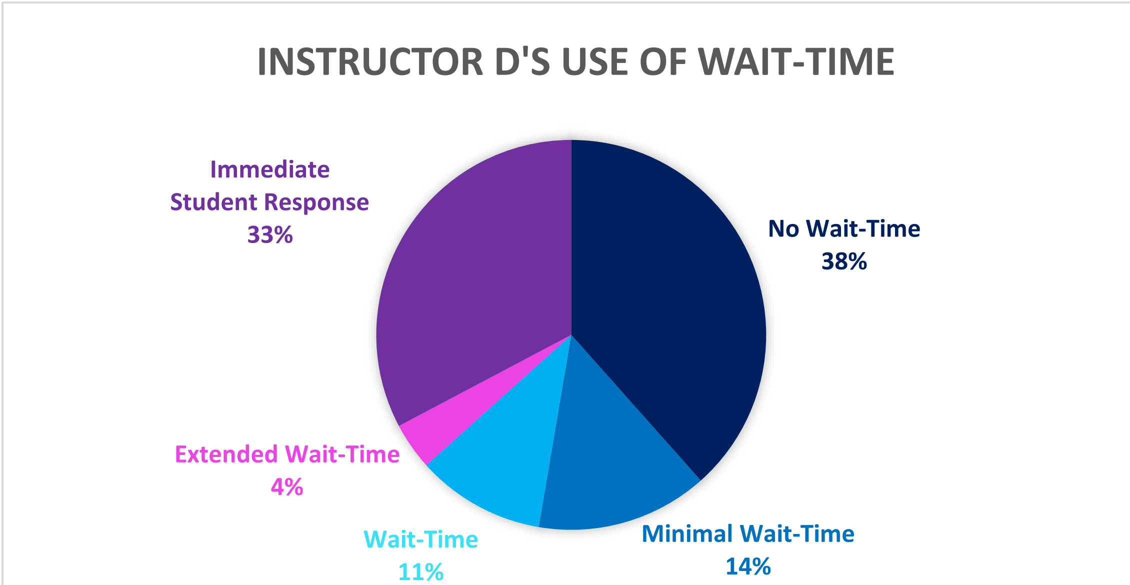

Figure 3: Instructor D’s use of wait time

A look at wait time per day, found in Figure 4, shows that various kinds of wait time were accounted for daily. In fact, six of the seven observed days had evidence of each wait time category; *Extended Wait-Time* was not present on Day 2. There was also proof that the students responded to Instructor D’s questions immediately on a daily basis. Some days this happened more frequently than others. All days except Day 3 she used *No Wait-Time* more often than she did *Minimal Wait-Time*, *Wait-Time*, and *Extended Wait-Time* collectively. Day 3 was different in that it was a mandatory review day in preparation for a test.

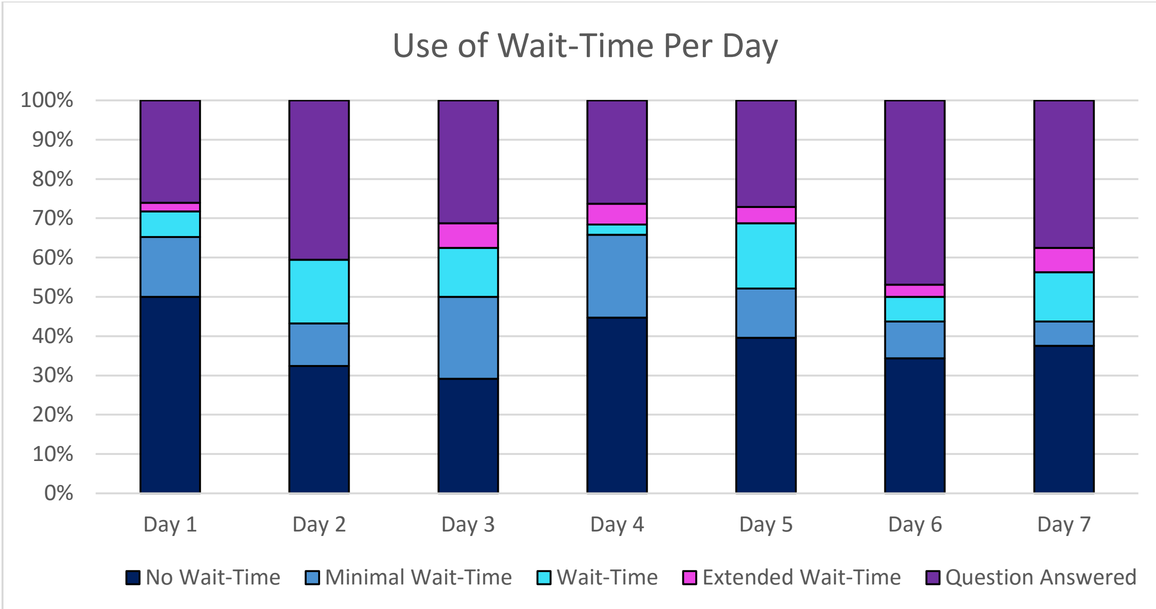

Figure 4: Instructor D's use of wait time per day

Instructor D did not utilize a particular kind of wait time for a specific question type as *Gathering Information*, *Generating Discussion*, *Requesting Validity*, and *Checking Understanding* questions were followed by each category of wait time. Some question types had low frequencies, yet all of them were either answered immediately, followed by *No Wait-Time*, or *Minimal Wait-Time.* This is observed in Table 9 which provides the frequency of each question type coded as *No Wait-Time, Minimal Wait-Time, Wait-Time, Extended Wait-Time*, or *Immediate Student Response.*

Table 9: Instructor D's Use of Wait Time by Question Type

	No Wait-Time	Minimal Wait-Time	Wait-Time	Extended Wait-Time	Immediate Student Response	Total
Gathering/ Leading	35	19	19	7	59	135
Generating Discussion	14	2	2	2	8	32
Requesting Validity	14	6	5	1	14	40
Checking Understanding	34	10	3	1	4	52
Meaning Relationships	2	0	0	0	0	2
Linking Applying	1	0	0	0	0	1
Orienting/ Focusing	1	0	0	0	0	1
Inserting Terminology	1	1	0	0	0	2
Probing	1	1	0	0	5	7
Extending Thinking	1	0	0	0	2	3
Establishing Context	4	1	1	0	0	6
Total	108	40	30	11	92	281

Questioning without wait time. Each question type present during Instructor D's observed sessions was at some point coded as *No Wait-Time.* The majority of these questions were followed by an instructor comment or explanation, but there was evidence of her asking back-to-back questions, and answering her own questions in less than two seconds. She promptly answered about 20% of the questions coded as *No Wait-Time*. The questions she answered were primarily of the *Gathering Information* type, representing questions the students should have easily been able to answer themselves. For example, she asked and answered the following: “The derivative of the bottom would be what?”, “Like what’s the area of a rectangle?”, and “What did you do if the powers were the same?” There were also single occurrences of Instructor D answering her own questions in five other question types, namely, *Orienting/Focusing, Probing, Generating Discussion, Context,* and *Linking/Applying.*

Questions with no wait time that were followed by an instructor comment or explanation typically went unanswered, yet on occasion a student answer was provided later. Consider the following example where a student answered the question after an instructor comment.

> I: Ok I’m seeing some of y’all hesitating plugging in the value. What’s my derivative? I think we all got that right.
>
> S: One third x raised to negative two thirds.

In a similar example, the instructor asked “What’s the derivative of x cubed?” A student answered the question after the instructor said, “This one you can do in your head.”

As previously stated, most of Instructor D’s questions that were followed by a comment or explanation went unanswered. These questions were mainly of the *Checking Understanding* and *Requesting Validity* types and could have been answered nonverbally. Questions like, “Does this make sense?” and “Are y’all all good with that?” asked students to examine their understanding of ideas but did not necessarily require a verbal response, particularly if they understood. Similarly, “Is that right?” and “You agree with that?” which asked students to validate claims did not necessitate verbal agreement.

Occasionally, questions were immediately followed by another question. This was not observed for a particular question type. The students never answered the first question when another question followed. They did, however, answer many of the subsequent questions. An illustration of this took place as Instructor D worked through a Newton's Method example and is in the excerpt below.

I: …"Ok so what would be a good decent first guess? Like what interval do I know that that intercept lies on just by looking at the graph? Let me stop, y'all look quizzical.

S: Negative pi over two to pi over two.

It can be seen that the student answered the second question, but it worth noting that the instructor ended up answering the original question herself after doing a bit of explaining.

Instructor D responded to her questions within two seconds on many occasions. Her response was generally a comment or explanation, but she also answered her own questions and at times followed them with another question. It was observed that the lack of wait time was not utilized with particular question types, as each question type had evidence of no wait time. These questions were sometimes later answered by students, however it was normal for questions with no wait time to go unanswered. This can be partly attributed to the instructor asking questions that did not necessitate verbal responses. Students were not limited to questions with no wait time because wait time was sometimes provided.

Questioning with wait time. Instructor D responded to her questions in high numbers when they were followed by wait time which is acknowledged here as *Minimal Wait-Time, Wait-Time,* and *Extended Wait-Time*. She responded to approximately 78% of her own questions, indicating that it was not the norm for students to respond when wait time was provided. Instructor D's responses were mainly comments or explanations similar to when there was no wait time given. Many of the questions were followed by another question, and again there evidence of her answering her own questions.

Unlike questions with no wait time, Instructor D only answered questions of the *Gathering Information* type when she used wait time. She asked and answered two questions after providing between three and five seconds of wait-time during an example using L'Hôpital's rule to find the limit of a function. The first question the students had already been allowed to consider as noted by her saying, "Let me give you guys a minute to take the derivative. So take the derivative of the top and the derivative of bottom. Be careful with the a…" She later said the following:

> …What's the derivative of one over x? Minus one over x squared and then one plus a over x, and then you'd have what negative a over x squared for the derivative? So ln of anything; one over the one plus ax, the derivative of the inside negative a over x squared the root of one over x, negative one over x squared. So we have b and then you have the limit as x goes toward infinity of let's see the x squareds would cancel; that minus would cancel with that minus, and so I'd have a divided by one plus a over x. And so what is that limit as x goes toward infinity? Just, that would just be a…

She answered her first and last questions which involved students' use of known rules and procedures, yet the second question represents an instance that she followed her question with an explanation after waiting three to five seconds.

It was observed that comments and explanations generally followed questions that did not require a verbal response. Consider the second question in the passage above, "…then you'd have what, negative a over x squared for the derivative?" This question was viewed as asking students for validation. There was no sign of verbal confirmation, yet the instructor paused three to five seconds before offering an explanation of the derivative of "ln of anything." In another example involving derivatives she asked, "Is everyone okay with the derivative?" This occurred after she talked students through finding two derivatives that involved using multiple rules. The question asked students to consider their understanding of what she had done, but again did not require a verbal response particularly if students were alright with it. She waited two to three seconds before going on to tell the students that she was "rewriting it" to simplify a little. Let it be noted that she may have been paying attention to nonverbal responses.

Other times the instructor asked a question, provided some amount of wait time, and after no student response she asked another question. In the example below she does this and then starts to explain to students what to do.

> …Ok so what form am I in right now? So which of these three forms am I illustrating first? So plug in kind of like what's happening if the x is 0. So the one plus $3x$ is tending toward 1…

Both of her questions were followed by three to five seconds of wait time. The instructor went on to explain before answering the question herself with "…So this is in the form one to the infinity…" This is one of few instances that the second question asked the same thing as the first. The instructor only answered her questions later than they were posed on seven occasions when wait time was offered. There was also very minimal evidence of students answering the instructor's questions at a later moment.

The students did not answer many of the instructor's questions when wait time was utilized. They delivered an answer to Instructor D's questions 19 out of 81 times, and two of them took place after some instructor response. Again, these questions were primarily of the *Gathering Information* type. In one particular case the instructor gave students a function f, and asked them to find the antiderivative with "What should be capital F?" A student responded "e to the x plus cosine x" after more than 5 seconds of wait time. The only other evidence of a student answer after some kind of wait time was in response to a *Probing* question that asked a student why he was taking the derivative and two *Generating Discussion* questions that invited students to share what they wanted to do during class (e.g. "Other ones you guys want to do?").

Instructor D asked many questions that were followed by wait times between 2 and 10 seconds. This represents time the students had to think and formulate answers, yet they did not typically respond. The instructor was the first to respond in most cases, but it was not always with the purpose of providing an answer. She responded with a question at a higher frequency than she answered questions. The response offered most often was a comment or explanation due to the nature of the instructor's questions; they did not always require a verbal response to be answered.

Summary. Questions during Instructor D's observed sessions generally received a response within a two second time frame. The responses mostly came from the instructor, but the students also replied quickly quite often. There was a noticeable number of questions that received some kind of wait time and like questions that got a response within two seconds, Instructor D was generally the first to respond. Her response was likely to be a comment or explanation regardless of if wait time was provided. Further, the instructor's use of wait time or lack thereof was not dictated by the type of question posed.

Classroom Setting

Several types of in-class activities took place during Instructor D's observed Calculus I sessions as seen earlier in Table 6. Be that as it may, the bulk of the class time was spent with the instructor transmitting information from the front of the room with little back and forth interaction with the students. The largest part of the instructor's questions were posed as she used lecture-discussion which makes sense as it was how much of the observed class time was used. It was during this particular in-class activity that she managed to ask questions of all 11 types.

Instructor D lectured when she introduced new material as well as when she reviewed concepts, yet there were interesting differences noticed in her questioning when talking about new versus previously learned material. First, she asked questions from a larger number of question types when lecturing about new material. Second, there was evidence of *Probing* with review but not with new material. Lastly, she provided wait time more often than she did not when she asked questions in the course of review; the only in-class activity this held true for.

The students did not spend a lot of time during class working on problems themselves because the instructor worked out most of the problems and talked students through them.

There were, however, a few occasions where the students had an opportunity to work individually and with a partner (see Table 6). Instructor D walked around as the students worked individually and in small groups, but she asked very few questions while she did so. She appeared to be spot-checking students' work at these times. This may have been because whatever the students were working on was later discussed as a whole group. The few questions asked as students worked in small groups were of the *Gathering Information* and *Requesting Validity* types. Similarly, these types of questions in addition to *Checking Understanding* were asked when students worked individually. An interesting finding in regard to the questions the instructor asked during individual work is that none of them received wait time. This was the only in-class activity where this held true. Additionally, individual work was the only in-class activity where *Gathering Information* questions did not hold the highest frequency. It is worth pointing at that there were only four questions posed during individual work, as such this may not have been the case with a larger number of questions.

The final observed in-class activity that Instructor D put to use was whole group discussion. This entailed the discussion of the students' individual and small group work, but it also included their discussion of homework because these were problems that the students may have previously tried or completed. Instructor D's questions were of the *Gathering Information, Checking Understanding, Requesting Validity, Generating Discussion*, and *Probing* types during whole group discussions.

The whole group discussions that followed the students' individual and small group work did not last long compared to the discussions on homework problems, which lasted the duration of a single class session. There were, however, more questions asked when discussing individual work and small group work. The discussions were also different in that the ones on homework were more like lecture-discussion since the instructor worked problems for the students, did a lot of explaining, and asked questions along the way. There was a little more than a three to one ratio of questions with no wait time to questions with wait time during discussions of individual or small group work. The wait time in discussions around homework were unique because they were evenly distributed; there were 16 questions immediately answered by students, 16 questions given some type of wait time, and 16 questions identified as no wait time.

During seven observed class sessions, the students in Instructor D's class were asked a wide array of questions in a setting that was dominated by lecture-discussion despite its inclusion of some individual work, small group work, and whole group discussion. The instructor asked the students questions of the *Gathering Information* and *Requesting Validity* types during each of the in-class activities, yet *Probing* only occurred when they were engaged in something the students had previously worked on. Particularly, *Probing* took place when they discussed homework problems, individual work, and small group work as well as when they reviewed. It was typical for the instructor to ask questions with no wait time within each in-class activity, but how it compared to her use of wait time differed per in-class activity.

Summary

Instructor D asked around 280 questions from 11 question types over the course of seven observed days. Many question types had low frequencies, and only three (*Gathering Information, Checking Understanding* and *Requesting Validity*) were used enough to produce percents over 10. Close to half of the questions *Gathered Information,* yet the range of types utilized indicates that she asked questions known to engage students beyond facts and procedures. Instructor D was generally the first to respond when she asked questions. She responded to more than a third of her own questions within two seconds, and was the first to respond to most of the questions given wait time. Her responses were comments and explanations most often, however, she also answered her own questions and asked back-to-back questions. The questions she answered were mainly of the *Gathering Information* type. The students answered a little over 30% of the instructor's questions within two seconds, but they answered minimal questions when wait time was provided.

A range of wait time was used during her questioning; lows less than a second and highs around 10 seconds. Wait time was used for each of the question types, but there were differences observed in wait time and question types based on different in class activities. Individual work, small group work, and whole group discussion were utilized occasionally in class sessions that were mainly lecture-discussion. Instructor D was observed providing wait time in each of the in-class activities with the exception of individual work, where she asked very few questions. It was common for her to not wait after asking questions, but she often paused for more than two seconds when she reviewed material. The review aspect of lecture-discussion was also one of two contexts she asked *Probing* questions. The only other time the instructor probed into students' thinking was during whole group discussion. This was not the only question type used exclusively in certain in-class activities evident by the difference in the number of question types used in lecture-discussion (11) and individual work (2).

Case of Instructor P

Instructor P's class met in a relatively small classroom that had around 20 desks with attached chairs. The desks were not organized in a particular layout. In fact, they were in different positions daily. There were, however, a few fixed objects in the room. The front wall had a dry erase board that was almost as wide as the wall itself. There was also a smaller dry erase board on a side wall in the back of the room. The smaller board was never used, but the large board was written on sparingly by the instructor each observed day. A podium with a computer sat at the front left of the room. Immediately beside the podium was a table that the instructor placed his materials on.

This was a small Calculus I class where five to eight students were in attendance during this study. Each class began promptly with the instructor asking the students if they had any questions or problems. The observed class sessions were similar in that the students were engaged in whole group discussion for the duration of each class. The instructor asked questions to get the students to think through problems that had either been assigned as homework or determined by the instructor himself.

Two main topics were discussed over the course of five days, namely, optimization and related rates. They worked together on at most four problems per class session. The operative words here are "worked on" because they discussed problems up to a point, and the instructor left the portion that involved solving using "mathematical manipulation" for the students to do outside of class. Remarkably, there was only one problem that the class talked through in its entirety. This was a rare moment as pointed out by the instructor himself when he said, "That's the first problem we've finished in all this class. I don't know why I did that. I'm feeling generous today I guess." Opposed to solving problems with their original numbers, they usually used alternative variables.

Instructor P did most of the talking due to the number of words in his talk turns compared to those of students. The bulk of the instructor's talk turns were under 100 words. Nevertheless, there were much larger talk turns observed each day. The instructor was generally explaining or setting the stage for problems using his life stories during longer talk turns. He also commented, asked, and answered questions. There was evidence of two extremely long talk turns that were right at 1000 words. The students were regularly engaged in discussions by the questions Instructor P asked despite the fact that he did most of the talking. An overview of each day can be found in Table 10.

Table 10: Overview of Instructor P's Class by Day

Day of Observation	Overview of Content	Classroom Context
Day 1	• Optimization	• Whole group discussion of problems
Day 2	• Optimization • Subtle introduction to antiderivatives	• Whole group discussion of problems
Day 3	• Optimization • Related Rates	• Whole group discussion of problems
Day 4	• Related Rates	• Whole group discussion of problems

Table 10 Continued

Day 5	• Related Rates • Optimization	• Whole group discussion of problems

Questions by Type

Instructor P asked a variety of questions during the observed sessions of his Calculus I class as seen in Table 11. Questions of the *Gathering Information* type were asked most often, however, questions of lesser frequency added to the instructor's overall questioning. The *Generating Discussion, Checking Understanding,* and *Requesting Validity* types attributed to more than a third of the instructor's questions. Additionally, *Probing* and *Meaning/Relationships* questions made up around 12% of his questions and gave students an opportunity to think more deeply. All other question types occurred at lower frequencies and will be given minimal attention. There was no evidence of the *Linking/Applying* category, as such it is not mentioned in the analysis.

Table 11: Question Types Used by Instructor P

Question Types	Frequency (Percentage)
1. Gathering Information	235 (46.3%)
2. Inserting Terminology	9 (1.8%)
3. Meaning/Relationships	30 (5.9%)
4. Probing	33 (6.5%)
5. Generating Discussion	59 (11.6%)
6. Extending Thinking	2 (0.4%)
7. Orienting/Focusing	1 (0.2%)
8. Establishing Context	5 (1.0%)
9. Checking Understanding	48 (9.4%)
10. Requesting Validity	86 (16.9%)
Total:	508 (100.0%)

Each of the observed days Instructor P utilized multiple question types. This is displayed in Figure 5 which shows the question types used per day. There were six questions types common to each day, namely, *Gathering Information*, *Generating Discussion*, *Checking Understanding*, *Requesting Validity*, *Probing*, and *Inserting Terminology*. Additionally, there were at least six question types used daily. Day 1 was unique in that it had evidence of all 10 question types. This section focuses on the findings within the question types that had the highest frequencies, but highlights all others at once in a smaller subsection.

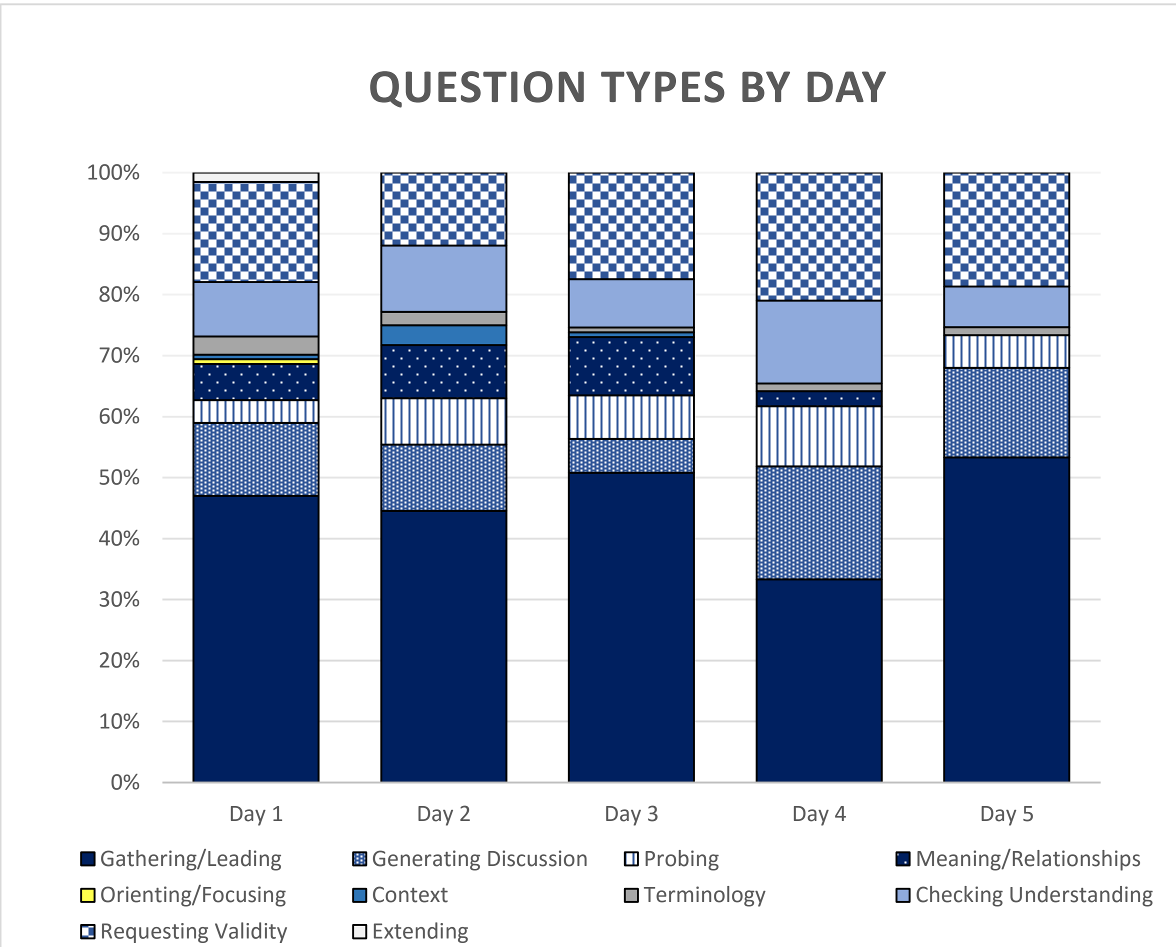

Figure 5: Instructor P's question types per day

Gathering information. Instructor P asked numerous questions that involved students' recollection of known facts and procedures as the class worked through problems together. *Gathering Information* questions were not all the same although they had this commonality. Some questions asked the students to state specific facts or procedures based on what they had previously learned. For example he asked, "How do you find maximums for a function?", "So how do I check rather a critical value gives me a maximum or not?", and "What's underneath the radical can't be negative and denominators can't be?" A couple of times students were asked to state a formula as demonstrated by "Does anybody know what the area formula is for a trapezoid?"

Other *Gathering Information* questions required that the students use known rules or procedures to perform computations. This showed up daily when the students were asked to find derivatives. Instructor P asked "So what's the derivative of x squared where x is a function of time?" and "If I take the derivative of pi minus theta with respect to theta of course, I'll get what?" for example. Three questions including "What's gonna happen with this top one?" were asked in regard to finding limits. There was also minimal evidence of questions that asked about numeric computations. Consider the following example as they turned revolutions per minute into radian measure while working on a related rates problem.

> I: …This thing is turning around here at two pi radians per minute and it's going to take 20 of those. That's going to give me what will that be, 20 times 2 is?
>
> S: 40

Additional examples involved students' use of known rules and procedures to find endpoints when an interval of optimization was not given.

Instructor P also used *Gathering Information* questions to ask students what to do next in a problem. One instance occurred as they worked on an optimization problem. Once they determined the function to be optimized, Instructor P asked “So what do I do?” A student responded “Take the derivative.” The instructor found the derivative, made a request for validity, and asked “So what do I do with that?” Again a student answered his question. Within this same discussion he asked, “And then how do you take a quotient and set it equal to zero?” and “How do I solve for x?” Similar questions were asked throughout the observed class sessions and were identified as such because they did not require anything more than the students sharing common knowledge.

Questions were asked daily that *Gathered Information* about the appropriate “models” needed to solve the optimization and related rates problem(s) of discussion. These questions also required students’ recollection of known formulas and rules. For example he told students he needed an “equational relationship between the two variables” written as they worked on a related rates problem involving a wagon being pulled. He asked how he could get that. A student responded “Pythagorean Theorem” which was based on her memory of right triangles and the theorem.

Instructor P occasionally asked students to extract given information from problems. These questions were easy to identify and included examples like “And it said height is what?” which was asked during a related rates problem about filling a conical tank and “The function they had was?” as they began an optimization problem. *Gathering Information* questions generally asked students for something that could be determined using known facts and procedures and often had answers that could been deemed correct or incorrect. The students answered these questions on a regular basis during each of the observed class sessions.

Requesting validity. Instructor P made daily requests for students to validate claims during this study. He mainly sought validation for the ideas he expressed himself, yet at times it was students' responses that were the center of validation. *Requesting Validity* questions were often in the form of statements followed by simple questions like "Right?" and "Isn't it?" There was, however, evidence of more explicit questions like "Did I do that right?" Like *Gathering Information* questions, the students verbally answered more than 50% of these questions. Their responses were generally in agreement (e.g. "Right" and "Yes"). In a couple of instances these questions sparked a student question, otherwise they may have been answered nonverbally. One example of Instructor P *Requesting Validity* of his claims took place on Day 1 as they worked on an optimization problem. They established that the minimum would have to occur at a critical point because the end behavior of the function indicated there was not a closed interval. Instructor P found the derivative of the function and stated, "So there's my derivative. And from what we know about the function it better be a zero for that derivative, hadn't it?" He was asking the class to validate an assertion that should have been common knowledge at this point based on their understanding of finding the minimum and maximum of functions. There was no evidence of a verbal response, yet the instructor went on to ask "What's the zero for that derivative?" as if someone had provided validation.

As the class worked on a related rates problem involving pouring water into a conical tank, Instructor P asked if anyone saw a way to get a relationship between the radius and the height. He called on a student to respond and the following discourse transpired.

S: Is r always going to be 2/3h? You're top formula you've got r is 2/3 of h.

I: Lu is telling us that r is going to be 2/3 of h regardless how they change. Right?

S: Yea.

I: How did you come up with that?

S: Um. Well because it said it's a cone, so I know your cone volume equation doesn't…

Instructor P asked Lu to validate the notion that he was indeed suggesting that "r is going to be 2/3 of h regardless..." This is one of many examples that a student answered, but it also represents an instance where the discussion continued beyond the requested validation with a *Probing* question.

Other examples of *Requesting Validity* involved statements followed by questions similar in nature to "Didn't we?", "Doesn't it?", and "Aren't they?" All of these questions gave students a moment to consider their agreement with that which had been stated or written regardless of the wording of the questions themselves. There was reason to believe that Instructor P expected students to answer these questions even if it was nonverbally because he was observed saying, "Ok Nathan agrees with me. I'm waiting for everybody to say ok" after a request for validation. It was not enough that one student had provided his verbal agreement. The other students must have provided nonverbal agreement because he continued to talk shortly thereafter.

Checking understanding. Students in Instructor P's class were also asked questions daily that challenged them to check their understanding of something. It was their understanding of problems at instants. The instructor told the students stories as a means of establishing problems a lot of times. When he did this he asked "Y'all see the problem?" and other variations to make sure the students understood what the problem was saying because they were not always written anywhere.

Instructor P was occasionally observed *Checking Understanding* after a student shared their ideas. An example taken as such was, "Y'all hear what Nathan's suggesting here?" The question was not literal in terms of the students hearing him, but asked if they understood what Nathan suggested. A couple of times he asked *Checking Understanding* questions in regard to his written work. For example, he rearranged an equation in a related rates problem on the board and asked "Everybody follow that alright?"

Additional questions of the *Checking Understanding* type were posed as the discussion of problems wrapped up. In one instance he asked, "Questions about this one?" after their discussion of a related rates problem involving a pulley and a wagon. A student responded, "Shouldn't the answer be negative." This is one of very few instances that a student verbally responded to questions of this type. The students only responded verbally when they had questions, and their silence in itself was understood as an answer. Instructor P was observed saying, "The silence says no" after asking a similar question. There was also occasional evidence of the students nodding indicating that some of these questions were in fact answered nonverbally. *Checking Understanding* questions gave students a chance to reflect on their understanding of problems and explanations and presented moments for further discussion when the students asked questions.

Generating discussion. Instructor P used *Generating Discussion* questions in two distinct ways to get his students engaged in their constant whole group discussions. First, he asked the students questions about what they wanted to do. He also asked questions to get the students to share their thoughts and ideas.

Instructor P made it a point to ask his students if they had any problems they wanted to discuss. What the students wanted to discuss was central as he was observed saying, “Particular problems? Let’s work your first then we’ll look at mine.” These questions were generally posed to the class as a whole, yet at times he called on students by name to see if they had any problems they wanted to look at (e.g., “Erin you got one?” and “Zoe anything that you run into yet that you want to bring up?”). These particular *Generating Discussion* questions are similar to *Checking Understanding* questions in that they require students to consider their understanding in order to answer the question. They were coded as *Generating Discussion* because they asked for contributions from the class that ultimately informed the focus of discussions. Students answered these questions at times, and when they did not the Instructor either selected a textbook problem or introduced one of his favorites.

The students were encouraged to share their thoughts and ideas as they discussed problems as a class. One example took place on Day 1 as they worked on finding the interval of optimization for a particular problem. Instructor P pointed out that finding the largest thing that x could be was harder than finding the smallest. A student verbally agreed with the instructor and the following transpired.

I: Help us out.

S: At first I thought it was three, but then I realized that y has to be greater than zero too, so if you solve for y and then you get y equals three halves minus x so x can't be greater than three halves because then y would be negative.

Here the instructor invited the student to share his thoughts about how large x could be. In another instance he said, "I see some of you are busy doing something. Tell me what you're doing." These questions were not asked as often, yet they encouraged students to make their thoughts public.

Probing. Questions identified as *Probing* asked students to further explain their thoughts and constituted around 7% of Instructor P's overall questioning. Various kinds of questions were identified within this question type. Some *Probing* questions involved the word "how". One example asked a student how he knew something. During an optimization problem the instructor asked for the zero of a particular derivative function. A student answered "one," and Instructor P followed that up with "Ok, Nathan gave us a right answer without any manipulation. How did you know one?" Another student was asked to express how she got two as the critical value while working on the same problem.

A couple of *Probing* questions utilized "why" and asked students to justify claims. Consider the excerpt below which came from their first related rates problem of the course about a sliding ladder. The passage begins with Instructor P simplifying the derivative.

I: … And what I get here then is that x times dx/dt is equal to minus y times dy/dt. Notice the minus. Does that make sense?

S: Yea.

I: Why does that make sense?

S: Because as y decreases the input gets closer to x; increases.

Here the instructor was asking the student to explain why it made sense for y times dy/dt to have a “minus” in front of it.

Other questions asked students to elaborate further than the answer they initially offered without the use of ‘how” or “why.” For example, Instructor P asked the class what would be constant in an optimization problem about finding a minimum cost to lay a pipe. A student responded that the “distance” would be constant. The instructor validated the response that it would indeed be a distance but asked, “What distance Montee?” so that the student could be more specific in terms of the distance he was referring to.

One other kind of *Probing* showed up, and it was in reference to meaning. For instance, the students were asked what 2 m/sec was in a related rates problem involving a pulley and a wagon. A student stated that it was “a velocity.” Again, Instructor P validated the response and probed further. He asked, “…, but what is it relative to this context?” and “What is it that’s changing with respect to time?” Based on the conversation that followed, it was gathered that Instructor P was trying to get the students to identify the symbolic representation of the given rate. *Probing* questions gave students an opportunity to think more deeply about the topic(s) of discussion regardless of what they probed students for.

Meaning/Relationships. Similar to *Probing* questions, *Meaning/Relationship* questions gave students an opportunity to engage in learning mathematics more deeply. These questions asked students to make meaning of ideas or make links between representations. Most examples took place during related rates problems. The students were generally asked to identify what was constant and what was varying in problems. This was taken as *Meaning/Relationships* because the students had to make meaning of verbal representations in order to establish what was changing and what was not in the given situations. They were also asked to explicitly interpret what constants or variables meant in the context of problems. Similar to other question types, the students answered the bulk of these questions showing their comfort in engaging in their class discussions.

Additional question types. *Inserting Terminology*, *Extending Thinking*, *Establishing Context* and *Orienting/Focusing* questions were posed at low frequencies during the observed class sessions. Together they accounted for approximately 3% of Instructor P's overall questioning. An example of each of the lesser used question types can be found in Table 12. The *Inserting Terminology* example is one of nine questions. There were five or less questions observed in the other question types. The *Orienting/Focusing* example existed in isolation.

Table 12: Examples from Instructor P's Low Frequency Question Types

Question Type	Frequency	Example
Inserting Terminology	9	Now this thing to use your book's language, is the what equation?
Orienting/Focusing	1	True but?
Extending Thinking	2	Could I get to it analytically?
Establishing Context	5	Anybody ever have this happen to them?

The *Inserting Terminology* example provided was in regard to the students using the phrase "constraint equation" during optimization problems. The instructor *Oriented/Focused* a student's attention to a restriction that was given on the board by saying "True but" after the student explained something that would have only been true if the restriction was not there. The student said "oh yea" and then verbally acknowledged the restriction. The students' thinking was *Extended* when Instructor P asked them "Could I get to it analytically?" This was asking them for an alternative approach. The *Establishing Context* example was in regard to a story Instructor P told about working on his house using a ladder, and the ladder began to slide. He asked students about their experience with this, and introduced related rates.

Summary. Close to half of the questions posed in Instructor P's observed class sessions *Gathered Information* from the students, however, he gave students different opportunities to engage in discussions. Some *Gathering Information* questions asked that students explain how to do things, but they were always based on a collection of facts and procedures that can be viewed as common knowledge to Calculus I students (e.g., solving equations). Instructor P determined what to do during class and asked students to share their thoughts with his use of *Generating Discussion* questions. He also dug into their thinking with his use of *Probing* and *Meaning/Relationships* questions. The students answered many of the questions asked; this includes nonverbal answers provided when the instructor *Requested Validation* and *Checked Understanding.*

Wait Time

The questions asked by Instructor P were followed by each wait time category as seen in Figure 6. The largest percentage of his questions were followed by wait time as *Minimal Wait-Time, Wait-Time*, and *Extended Wait-Time* collectively followed 44% of the instructor's questions. Many of the instructor's questions yielded a response within 2 seconds, but this was not indicative of a lack of wait time because it was the students who responded most often to the instructor's questions during this time. The instructor responded within two seconds to a low 14% of his questions.

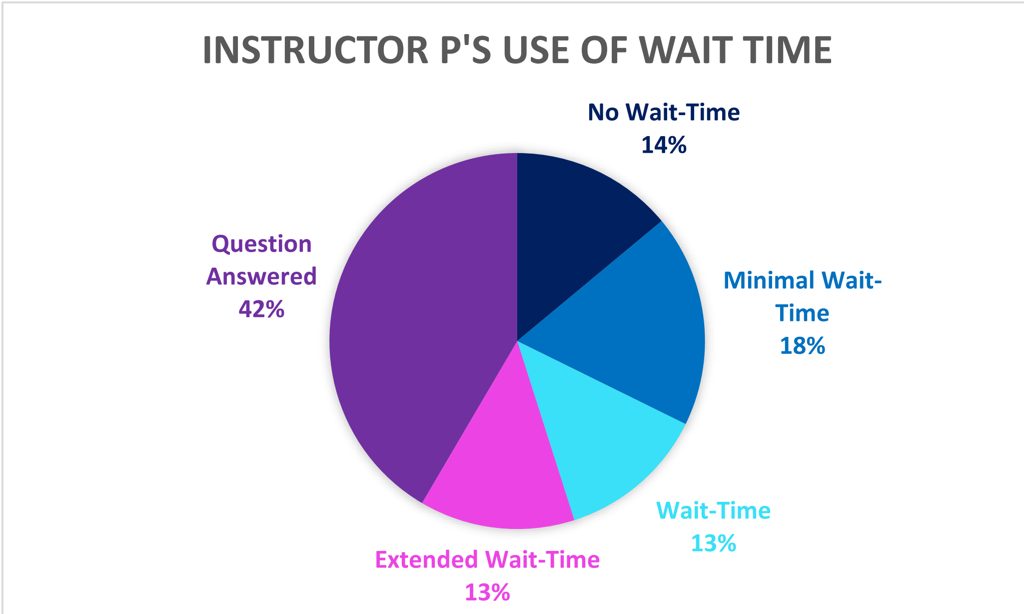

Figure 6: Instructor P's use of wait time

A look at wait time by day, found in Figure 7, shows that each of the wait time categories were present each day. It can be extracted that the instructor's questions were followed by wait time more often than not on each observed day. Additionally, the students regularly responded within two seconds and did so more often than the instructor did. Day 3 stands out because *Extended Wait-Time* had the highest percentage on that day, and questions followed by wait time as a whole were dominant.

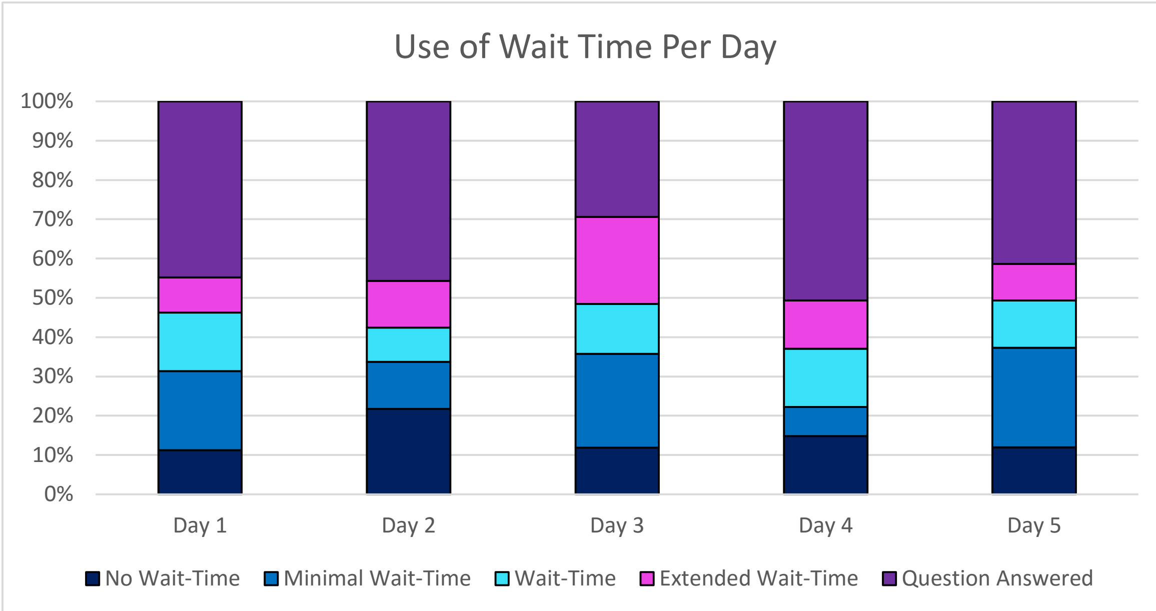

Figure 7: Instructor P's use of wait time per day

Instructor P utilized various wait times for the different question types asked as seen in Table 13. Five question types, namely, *Gathering Information*, *Generating Discussion*, *Checking Understanding*, *Meaning/Relationships*, and *Probing* had evidence of each wait time category. Moreover, all questions types with the exception of *Orienting/Focusing* were followed by *Minimal Wait-Time, Wait-Time,* and/or *Extended Wait-Time* during this study. There was a single question that was answered immediately by a student in this type. The remainder of this section focuses on describing the instructor's questioning with and without wait time in more detail.

Table 13: Instructor P's Use of Wait Time by Question Type

	No Wait-Time	Minimal Wait-Time	Wait-Time	Extended Wait-Time	Immediate Student Response	Total
Gathering Information	27	47	30	27	104	235
Generating Discussion	12	4	5	16	22	59
Requesting Validity	0	21	15	9	41	86
Checking Understanding	21	10	8	6	3	48
Meaning/ Relationships	4	3	4	7	12	30
Orienting/ Focusing	0	0	0	0	1	1
Inserting Terminology	0	2	0	0	7	9
Probing	5	4	2	3	19	33
Extending Thinking	1	1	0	0	0	2
Establishing Context	1	1	1	0	2	5
Total	71	93	65	68	211	508

Questioning without wait time. Seven question types yielded no wait time at some point during the observed classes. This included the following types: *Gathering Information, Checking Understanding*, *Generating Discussion, Probing, Meaning/Relationships, Extending Thinking,* and *Establishing Context.* The bulk of these questions were of the *Gathering Information* and *Checking Understanding* types. Instructor P's response was generally a comment or explanation, yet he answered 25% of the questions followed by no wait time. There was also minimal evidence of him asking a subsequent question as well as acknowledging a student to respond.

The questions that the instructor answered were mainly of the *Gathering Information* type representing questions the students should have been able to answer themselves. For example, he asked and answered “And then how do you take a quotient and set it equal to zero?”, “What’s the derivative of the right hand side?”, and “Let’s see a half of 9.8 is what?” There were four instances of the instructor answering his question after first making a comment or explanation. One example took place as they worked through an optimization problem. He said, “That is from here I’m gonna get what? Let me see, y’all make sure that I do the algebra right. So l is going to be equal to p minus w plus π times w, divided by two.” Here Instructor P was asking the students how they would get the constraint equation, p, solved for l. He paused less than two seconds before telling the students to make sure that he did “the algebra right.” He then answered his question aloud.

The instructor’s questions that were followed by comments or explanations were at times *Checking Understanding* questions that could have been answered nonverbally. This included but was not limited to examples like, “Questions about this stuff?” and “Julia is this alright with you?” The instructor could have easily been paying attention to head nods or other nonverbal gestures. *Generating Discussion* questions that asked students if there were any particular problems they wanted to look at were also followed by an instructor comment or explanation at times. Again, the students may have nonverbally responded. The same thing was true for the single *Establishing Context* question followed by no wait time which asked

“Any of you ever heard of Tyler Simpson?” Again, silence itself may have been understood as an answer.

As previously mentioned, the instructor went on to answer minimal questions after first commenting or explaining. There was also evidence of the students answering Instructor P’s questions after he first commented or explained. Let it be noted that the students did this twice as often as the instructor. Consider the short excerpt below.

> I: Does anybody know what g is? It’s a number of course it’s a fixed constant.
>
> S: 9.8

Here a student responded after the instructor stated that g was a number. The students knew what the instructor was asking for because they had previously pointed out that g was gravity.

Minimal questions with no wait time were followed by another instructor question. For example, “Anybody got anything else? Shantha you got one?” The first question was posed to the entire class, and in less than two seconds Instructor P decided to ask a particular student. Another example took place as they worked on an optimization problem. The instructor asked:

> Anybody else see a need for anything else that we would have or need that we would need to know before we tried to solve the problem? Alright, now what’s the variable or what are the variables if you want to put more than one? Lu.

The instructor asked his second question less than two seconds after asking the first one, yet the students may have responded nonverbally. This example also shows the final way that Instructor P quickly responded to his questions and that was by acknowledging or calling on a student to answer his question. This was seen when he called on Lu within two seconds of asking, “Now that’s the variable or what are the variables if you want to put more than one?”

Although questions were followed by no wait time daily in Instructor P’s class, they were of low occurrence. Instructor comments and explanations were the normal response to these questions, and there were times that he answered his own question immediately or after first commenting or explaining. The students were also observed answering questions after Instructor P commented or explained. Occasionally, there were back-to-back questions. As previously noted, there were many questions that could have been answered with gestures.

Questioning with wait time. Instructor P used wait time regularly when his questions were not immediately answered by students. *Minimal Wait-Time* was observed most often, yet approximately 30% of the questions identified as having wait time consisted of pauses of more than five seconds in alignment with *Extended Wait-Time*. The instructor and the students responded at similar frequencies, however, the students actually answered questions at a frequency higher than the instructor himself. Like questions with no wait time, the instructor’s typical response was a comment or explanation. There was also evidence of him asking back to back questions.

Very few questions were answered by Instructor P when some form of wait time was provided. Similar to the questions he answered with no wait time, these questions were mainly of the *Gathering Information* type. For example he asked, “What’s the log of one?” paused between two and three seconds and answered “zero.” In another example he waited three to five seconds before answering. Consider the excerpt below.

I: …What is velocity? [>5 seconds]

S: The derivative.

I: Velocity is the first derivative. Right? [>5 seconds] How do you find the maximum of a function? [3-5 seconds] Take its derivative…

The instructor asked three questions during this passage; all of which were followed by wait time. The first question represents one of the many instances a student answered the instructor after more than five seconds of wait time. The last two questions exemplify back to back questions separated by wait time. Instructor P answered the final question himself after waiting three to five seconds for the students to respond.

Many times instructor comments and explanations followed a question that the students may have answered without verbally speaking. This included *Checking Understanding* questions similar to "Y'all ok so far?" and "Questions?" as well as *Requesting Validity* with statements followed by questions like "Right?" and "Can't I?" There was also evidence of this with *Generating Discussion* questions around what problems to discuss. Instructor P commented and/or explained after questions of other types as well. Aside from the aforementioned types, these questions were mainly *Gathering Information.* One example took place after he asked a *Gathering Information* question about the formula for the volume of a cone during their discussion of a related rates problem. After pausing between two and three seconds he said:

> That one I can remember. I'll tell you how I remember that. The volume of a cylinder is pretty easy to visualize and so it goes around this way, pi r squared, and it's got a height and the volume of a cone is 1/3 the height of a cylinder with the same radius and height, so I actually remember this 1/3 to figure this one out.

The instructor first commented. He then went on to explain how he remembered the formula. At the same time he was writing on the board, so he also answered his question.

The students went on to answer his questions on a few occasions. This is evident the excerpt below which also took place during a related rates problem.

> I: Ola, Ves, do one of you guys see the relationship? You want to do it with angles but I'm going to do it just with distances here.
>
> S: l over, would you do s plus x?

Ola continued to answer after Instructor P validated her response. In another example, the instructor asked his students if there were any other problems they wanted to do before they worked on the problems he desired to suggest. He said, "Ok Celethia, any others?" and paused for two to three seconds before saying "I see Celethia drawing a picture there." She then brought up a problem she had been looking at.

Similar to questions with no wait time, the instructor periodically asked back-to-back questions. These questions were mainly of the *Requesting Validity* and *Checking Understanding* types. It has been stated numerous times that these questions could have been answered nonverbally with examples being "So if I go right in here and look at that angle, it is pi minus theta. Right?" and "So you all see where Shantha got the four over four which is one?" respectively.

Wait time was a regular occurrence in Instructor P's observed classes, and it was something he did deliberately; waiting for students to respond and giving them time to think. This was taken from a couple of comments that were made in addition to the length of pauses that were identified as *Extended Wait-Time.* One comment was made after he waited approximately 20 seconds for a student to answer a question. He said, "Remember I'm a Quaker I'm used to silence. That's not a rhetorical question." Another comment was made after asking the students if they saw a way to get a relationship between variables in a related rates problem. He said, "Now while you all are thinking about that let's do one other thing…" after seven seconds of wait time. Astonishingly, there were wait times of more than 30 seconds. Examples of such can be found in Table 14 with the corresponding question.

Table 14: Instructor P's Use of Wait Time beyond 30 Seconds

Question	Wait Time (in seconds)
Now how do I get t in terms of theta?	42.63
Anybody got any ideas?	33.08
What do I put over here?	36.40

Summary. Wait time in Instructor P's class was not an isolated event. It was utilized daily with various question types and regularly came before a student answer. *Minimal Wait-Time* was observed more frequently than *Wait-Time* and *Extended Wait-Time*, yet questions coded as *Extended Wait-Time* entailed pauses over 30 seconds. The students regularly responded to the instructor's questions both immediately and when wait time was provided. The instructor did not answer many of his questions, but he did so more often when no wait time was provided.

Classroom Setting

Whole group discussion was the sole in-class activity observed during this study in Instructor P's Calculus I class. Due to this fact, the instructor's questions and use of wait time cannot be discussed in terms of different in-class activities. Each day the instructor asked the students a variety of questions about problems that were either selected by the students or the instructor himself.

Summary

Ten question types, namely, *Gathering Information, Generating Discussion, Checking Understanding, Requesting Validity, Probing, Meaning/Relationships, Establishing Context, Extending Thinking, Orienting/Focusing,* and *Inserting Terminology* were captured in approximately 500 questions in Instructor P's observed Calculus I sessions. His questions were primarily of the *Gathering Information* type, yet engaging students in discussions was a focus. The students were given opportunities to explain with *Probing* and *Meaning/Relationships* questions (about 11%), but some *Gathering Information* and *Generating Discussion* questions presented moments for students to explain as well.

Each of the question types were followed by wait time at some point with the exception of *Orienting/Focusing.* This was not by coincidence as Instructor P used wait time noticeably more than no wait time and consciously did so. Instructor P answered his own questions minimally, but commonly responded with a comment or explanation regardless of if wait time was given. The questions he answered were mainly of the *Gathering Information* type. In many cases, those followed by a comment or explanation could have been answered nonverbally. The students answered around 40% of the instructor's questions within two seconds and around half of the questions followed by wait time. They were also given a chance to answer questions with no wait time when Instructor P acknowledged them by name. All of the questions were posed during whole group discussion; the only in-class activity utilized during this study.

Case of Instructor F

Instructor F's Calculus I class met in a lecture hall organized in eight rows, each with between 6 and 10 seats. There was a large dry erase board on the front wall, with a smaller dry erase board to the right of it. The large board took up a hefty area of the front wall because it was stacked; meaning there were multiple dry erases boards that could be slid up and/or down. A table sat at the front of the room where the instructor kept her materials.

Class began promptly with an average of 29 students in attendance daily during this study. Each day the students could be seen writing as the instructor spoke and wrote detailed notes on the board. On an average day the instructor used lecture-discussion from the start of class to the end using the prewritten notes she held in her hand. This was not true on Days 2 and 3 where a small portion of class time was devoted to a quiz. Additionally, a few minutes were spent on these days discussing the quiz as a class.

The students in Instructor F's class did not spend any time working on problems with their peers, as such they only spoke if the instructor asked a question and when they had a question or comment of their own. The instructor did a lot of talking and often spoke in talk turns over 100 words. There were quite a few instructor talk turns over 300 words; two of which exceeded 1000 words. During these talk turns the instructor explained, commented, and asked and answered questions around a number of topics. An overview of the observed class sessions can be found in Table 15.

Table 15: Overview of Instructor F's Class by Day

Day of Observation	Overview of Class Session	Classroom Context
Day 1	• First derivative and curve sketching • Second derivative and curve sketching	• Lecture-discussion
Day 2	• Review of increasing/decreasing and critical points using first derivative • Quiz • Second derivative and curve sketching	• Lecture-discussion • Whole group discussion about quiz
Day 3	• Review of first & second derivative and curve sketching • Quiz • Optimization	• Lecture-discussion • Whole group discussion about quiz
Day 4	• Optimization • Review of implicit differentiation	• Lecture-discussion
Day 5	• Related rates	• Lecture-discussion

Questions by Type

There was evidence of all 11 question types in Instructor F's observed class sessions as seen in Table 16. An overwhelming 72.2% of the instructor's questions were identified as *Gathering Information*. Six question types (*Probing, Checking Understanding, Requesting Validity, Generating Discussion, Orienting/Focusing,* and *Meaning/Relationships*) had double digit frequencies, yet collectively they only constituted a little over 25% of the instructor's questions. Despite relatively low frequencies, the students were asked questions that allowed them to think about mathematics beyond procedures.

Table 16: Question Types Used by Instructor F

Question Types	Frequency (Percentage)
1. Gathering Information	524 (72.2%)
2. Inserting Terminology	5 (0.7%)
3. Meaning/Relationships	15 (2.1%)
4. Probing	48 (6.6%)
5. Generating Discussion	34 (4.7%)
6. Linking/Applying	8 (1.1%)
7. Extending Thinking	3 (0.4%)
8. Orienting/Focusing	14 (1.9%)
9. Establishing Context	2 (0.3%)
10. Checking Understanding	41 (5.6%)
11. Requesting validity	32 (4.4%)
Total:	726 (100%)

A look at the instructor's use of question types by day, found in Figure 8, shows that *Gathering Information* questions dominated the instructor's questioning on four of five days. It can be seen that six to nine question types were present on each day. Days 1 and 5 had the most question types present, yet the questions on Day 5 Instructor F pushed the students deeper than any other day with more evidence of the *Probing* and *Meaning/Relationships* types. This section gives attention to all of the observed question types; however, the focus is on those that are common to each day, specifically, *Gathering Information, Checking Understanding, Generating Discussion, Requesting Validity,* and *Probing.*

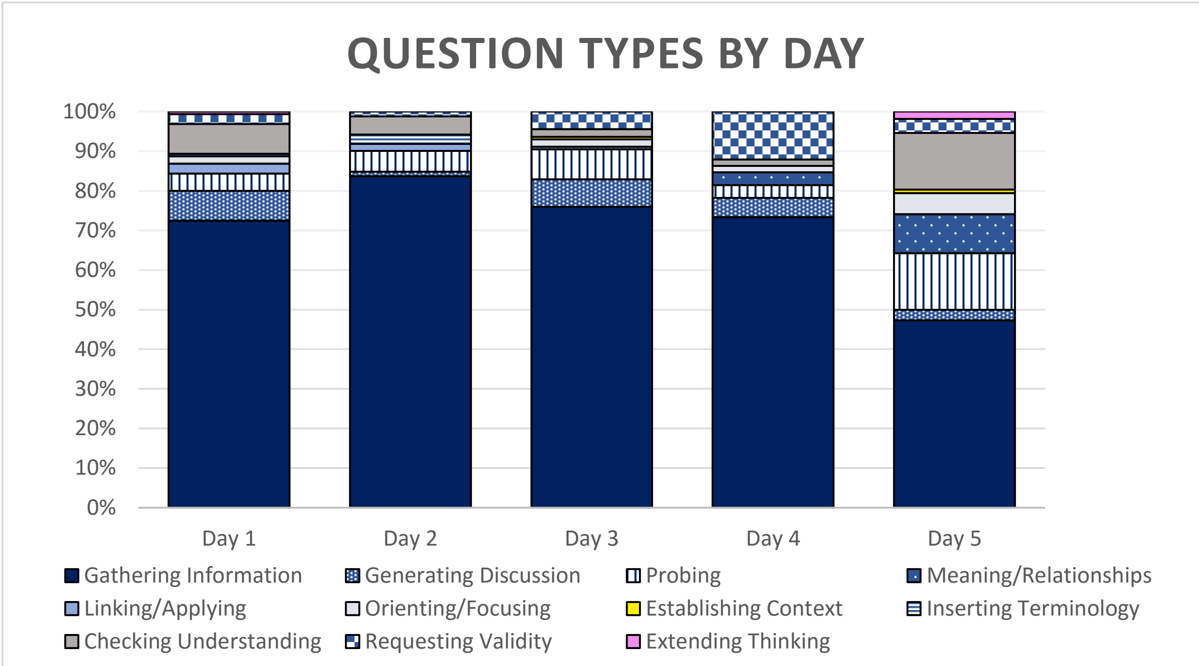

Figure 8: Instructor F's question types per day

Gathering information. Instructor F *Gathered Information* from her students on a regular basis using questions that required students' recall of facts or procedures. This was observed immediately on Day 1 when the instructor drew the basic sine curve on the board and asked the students a string of questions about the key x-values as well as where the function is increasing, decreasing, and constant. Below is part of the described dialogue.

> I: Where is this function increasing and decreasing? Well the first thing you're going to have to remember is what these kind of special points are corresponding to this graph. For example, what is this x-value here?
>
> S: Negative pi.
>
> I: Negative pi. And what is this value here?
>
> S: Negative pi over two.
>
> I: Negative pi over two. I'm not going to mark it. What's this right here?

S: Pi over two.

I: Pi over two. What's this one here?

Similar questions were posed until the remainder of the key x-values values were stated. The instructor then asked questions like, "So where is this function decreasing?", "What does it mean to be decreasing?", and "Where's it increasing?"

The instructor also asked questions that required that the students recall the theorems they were exposed to during class time. On Day 2, for example, they re-established that the first derivative of a function is positive when the function is increasing and negative when it is decreasing. Additionally, the second derivative of a function is positive when a function is "exactly concave up" and negative when it is "exactly concave down." The instructor drew four "basic shapes" on the board and the following series of questions transpired.

I: …So now let's come back to here. Let's look at first and second derivative. So in this type of shape is the first derivative positive or negative?

S: Positive.

I: This one here. It's positive. Ok. Second derivative, is it positive or negative?

S: Negative.

I: Negative. Very good. What about this shape here? f the first derivative. Positive or negative?

S: Negative.

I: Negative. Second derivative. Positive or negative?

S: Positive.

I: Um. Positive. Very good. Over here. The first derivative. Positive or negative?

S: Positive.

I: Positive. Second derivative is positive or negative?

S: Positive.

I: Positive. Very good. Last one here. First derivative positive or negative?

S: Negative.

I: Well actually, I drew it so bad, it looks almost not differentiable. Let's make it like that. What's the second derivative?

S: Positive.

S: Negative.

I: Negative. It's concave down…

The students answered these questions with responses that showed their recollection of theorems and received validation from the instructor for doing so.

Instructor validation was also seen with *Gathering Information* questions that focused on computation. For example, the students mentally performed computations as the instructor led them through creating sign charts that she called "portraits" that showed where the derivative values were zero, undefined, positive, and negative. In one instance she wanted the students to determine the sign of $f'(10)$ "without checking the actual value." She asked "…is this positive or negative?", "Is this negative or positive, the denominator?", and "So is the whole thing positive or negative?" In another instance, she asked students to compute function values in the process of determining the minimum and maximum of a function. The students were asked to find $f(1), f(0), f(-1),$ and $f(4)$ for one problem.

Other computations involved using known rules to find derivatives. Examples of the questions posed include the following: “What’s the derivative of e^x?”, “y prime is what?”, “So what’s the derivative of b with respect to g?”, and “So what is da/dt?” Aside from finding derivatives, students were sometimes asked to identify the rules that would be needed to do so. Consider the following the excerpt.

> I: …What is the derivative of s with respect to t now? So now I have to look for this thing. What kind of rules are going to appear there?
>
> S: Product rule.
>
> I: Product rule for sure. I have a product here. What else?
>
> S: Power rule.

A few times the instructor asked questions to lead students through using a particular derivative rule. This was observed with the product rule as they worked on an optimization problem. Instructor F said, “If I apply the product rule to this expression what’s going to be the first term here?” and “And what’s the second term?” Prior to this she asked what the rule said which asked students to simply state the rule.

There was also evidence of *Gathering Information* questions that either asked students what they did in a previous class or asked for procedures or next steps. Specifically, Instructor F asked “What did we talk about the last time” and similar questions. She also asked questions like “Ok so what was the procedure for optimization of a function?” and “Ok, so what did we learn that about these examples again in terms of, when we do have a minimum or maximum value?” *Gathering Information* questions were typically answered by the students. The answers were short responses and at times they were followed up by *Probing* questions discussed in the next section.

Probing. Questions identified as *Probing* had the second largest frequency of the observed question types (around 7%). *Probing* questions followed-up on students' thinking in different ways. Many of these questions asked students "Why?" and mostly asked them to justify their claims. Consider the excerpt below, which occurred as the instructor guided the students through using the first and second derivative tests to sketch the graph of a function.

I: ...Is this a local minimum or maximum or I don't know?

S: Minimum.

I: It's a local minimum. Why is it a local minimum?

S: Because the derivative is greater than zero.

I: Because the second derivative is greater than zero…

A student was asked to justify why the critical point, x=1, was a local minimum in this particular passage. The instructor asked a similar question in regard to why zero was an inflection point in a different problem. In another example a student was asked to verify her claim that the statement "If f has a critical point at f equals -1, then it has a local min or max" is false.

Instructor F *Probed* students by asking questions that required that the students elaborate further at times. The passage below provides one example that took place as they talked about curve sketching.

I: … And now what am I going to have on this third piece?

S: Positive.

I: Well who's positive?

S: The first derivative.

Here the instructor was asking the student to elaborate by asking “who’s positive.” In another instance, the instructor asked a student “the area of what?” There was minimal evidence of other kinds of *Probing* questions like “How did you do this?” which asked a student to explain their approach and “Ok, but what does this mean in the context of this problem?” which *Probed* for meaning. All of the *Probing* questions observed gave students a chance to explain their thinking. In most cases they were answered by the students which suggests that the students took advantage of these opportunities. Similar to when the instructor asked *Gathering Information* questions, she often validated the students’ answers by repeating them before moving forward.

Requesting validity. Students in Instructor F’s class were asked to validate claims on each observed day. These requests made up less than 5% of the questions the instructor asked. *Requesting Validity* questions were generally easy to identify because they asked students if they agreed with something that had been verbalized or written. The students answered these questions as often as they did not with both agreement and disagreement.

One example of *Requesting Validity* took place on Day 1 as the instructor led the students through determining where $h(x) = \frac{x^2+10x+1}{x-2}$ is increasing, decreasing, and constant, as well as where it has a local minimum and maximum. After they worked to find and simplify the derivative of $h(x)$, Instructor F explained:

> …I need to know where it is zero. I need to know where it is undefined. I need to know where it is positive and I need to know where it’s negative. Do you agree? What will it be called when I have this derivative is zero or undefined?

Here she was asking the students to validate her claim that she needed to know where the derivative was zero, undefined, positive, and negative in order to complete the task at hand. The instructor followed this request for validation with another question after no verbal response from the students. There were only three other *Requesting Validity* questions that were followed by an instructor question. The example in the excerpt above is the only example where the subsequent question was not another request for validation.

Other questions of this type asked students to validate the instructor's written work. On Day 1, for example, Instructor F led the students through finding the derivative of a function using the quotient rule and the dialogue below followed.

> I: Ok, so now I'm going to expand this. And I have $4x$ squared minus $10x$, 12, so $10x$ minus four is $6x$ minus 20 minus x squared plus minus $10x$ minus one over x minus two squared. Is that right?
>
> S: No.
>
> I: No?
>
> S: Wouldn't it be 2x squared?
>
> I: Umm. Ok, so I have 2x squared that's good, 2x squared very good…

It can be gathered that a student voiced his disagreement with what was written on the board and was allowed to express what he thought was wrong. Similarly, Instructor F asked "Now before I continue can you just check for me that I applied that law correctly?" on Day 5 after she had written a proportion on the board using the "the law of similar triangles." There was no verbal response in this case, but there is reason to believe that validation was expected even if it was nonverbal because of the wait time provided. It is worth pointing out that the students were asked to validate their own claims in a couple of instances. Instructor F asked other questions that *Requested Validity* like, “Did I make a mistake?” and “Is this true or false?” There were also statements followed by “Right?” which too, *Requested Validity* from the students.

Checking understanding. Questions of the *Checking Understanding* type represented less than 6% of Instructor F’s overall questioning. Roughly half of the occurrences clearly asked the students if they had any questions. The instructor used this question type at various points before moving forward in her class sessions. An example lies in the following excerpt.

> I: At one. The global minimum value is negative one and it happens at x equals?
>
> S: Four.
>
> I: Four. Very good. Ok, so the procedure is find the critical points, looks at the values, and then take the biggest and the smallest. Ok, questions?

This passage came from the end of the dialogue that transpired in regard to finding the global minimum and maximum of a piecewise function on a closed interval. The instructor reiterated the procedure for optimization and asked students if they had “Questions?” before moving into optimization problems with real-world contexts. The students were also asked if they had questions on Days 2 and 3 after the instructor reviewed material with the students in preparation for a quiz. For example, she asked “Any questions?” on Day 2 before saying “Ok let’s take a quiz.”

Checking Understanding questions took other forms during Instructor F’s class sessions. She was observed asking “Does this make sense?”, “Does that make sense?”, and

"Is everybody clear on this?" as she offered explanations and copied notes on the board. Another variant of this question type can be found in the selection below.

> I: …So I have $s + l$ or l, that's this side and for the other one I have the vertical it's 10 or 6. So now I'm going to simplify that so from here we have that multiplying by 60 on both sides I get that $6t$, $6s(t)$ plus $6l(t)$ is equal to $10l(t)$ and that means that $6s(t)$ is equal to $4l(t)$ and I'm left with $s(t)$ is $2/3l(t)$. Ok, so is everybody with me so far?
>
> S: You lost me right there.

The instructor stopped while working on a problem to have students check their understanding of what she had done up to that point by asking if everyone was "with" her. Notice that a student replied to her question suggesting his lack of understanding. This led to further explanation by the instructor and another opportunity for students to examine their understanding when she later said, "Ok, other questions?"

She generally asked if there were "other questions" after she addressed a previous student question or concern. Consider the excerpt below, which took place after she finished a different related rates problem.

> I: …Ok, so questions about that? Yea.
>
> S: So that's the answer?
>
> I: Well let's see. Unless I made a mistake which is always possible. So first thing is am I looking for, am I looking for this rate, am I looking for dr/dt?
>
> S: Yea.

I: Yes. Ok good. So then maybe I may have made some you know when I divided I multiplied or something like that, but assuming that then that's the answer. Ok? Other questions?

S: Is r of t squared same as r squared t?

The students were asked to check their understanding two times in this passage, and they responded both times. In most cases the students did not respond to questions of this type, but when they did it was because there was something they were unsure of. This suggests that the students may not have found it necessary to verbally respond to *Checking Understanding* questions when they understood or followed what was going on. It can be gathered that these questions not only gave students a chance to examine their understanding of the mathematics at hand, but they produced further discussion when students expressed questions or concerns.

Generating discussion. Questions identified as *Generating Discussion* were not observed in large numbers, but they identify moments that the instructor could have given students to express their ideas about what things mean or how things could be done. The instructor asked questions like, "What is a portrait of f prime?" and "What's related rates?" but answered them, so there was no indication of what the students may have known. She also asked questions similar to "What should I do here?" to get students to share their ideas about how to approach a situation. At times the students answered these questions, other times they were answered by the instructor. Additional questions that were understood as *Generating Discussion* were subtle in asking for contributions from other students and occurred less frequently. For example "What would be another example?" The questions did not generally produce discussion that entailed much student talk, and at times appeared to be questions the instructor intended to answer herself as a part of her lecture.

Additional question types. Instructor F used questions in the *Establishing Context, Inserting Terminology*, *Orienting/Focusing*, *Meaning/Relationships*, *Linking/Applying,* and *Extending Thinking* types minimally. These types together constituted less than 10 % of the questions the instructor asked over the course of five days. All of them had a frequency less than 10 with the exception of *Orienting/Focusing* and *Meaning/Relationships.* It is important to give attention to these questions, despite their low occurrence, because they help paint a picture of Instructor F's questioning. An example of each question type can be found in Table 17. Most of the example questions provided were answered by students.

Table 17: Examples from Instructor F's Low Frequency Question Types

Question Type	Frequency	Example
Establishing Context	2	Ok, so what does a balloon look like?
Extending Thinking	2	So a separate question which is not part of this might be, what is dr/dt; could I answer that question?
Inserting Terminology	5	What will it be called when I have this derivative is zero or undefined?
Linking/Applying	8	So what would be an example of something that was having a constant piece?
Orienting/Focusing	14	What am I trying to do?
Meaning/Relationships	15	So what is dr/dt?

The *Establishing Context* example was asked as they prepared to discuss a related rates problem about inflating a balloon. Similarly, the *Extending Thinking* example came from the lesson on related rates. The instructor *Extended* the situation beyond what the problem asked for and asked if that question could be answered. The *Inserting Terminology* question was explicit in its function to get students to use appropriate terminology. *Orienting/Focusing* was also explicit in that the instructor was focusing the students' attention on what was being asked. The *Linking/Applying* example was identified as such because the instructor was asking the students to apply their knowledge by providing an example. The students were given an opportunity to make *meaning* on a couple of days, but the most prevalent was with related rates problems. The example provided was not asking the students to find dr/dt, but was instead asking them what it meant in the context of the problem.

Summary. The questions posed in Instructor F's observed classes were mainly of the *Gathering Information* type (around 72%), yet there was evidence of all 11 question types. It was recognized that the instructor used *Gathering Information* questions to collect different information. Even with the large number of *Gathering Information* questions there was evidence of the instructor asking questions that engaged students beyond known facts and procedures (e.g., *Probing*, *Meaning/Relationships)*. The instructor *Requested Validity* of claims with her questions, and she regularly validated students' answers. There was a lot of instructor explaining, as such she also gave students moments to check their understanding of that which had been said or done.

Wait Time

Instructor F's questions were followed by various kinds of wait time during the observed class sessions as seen in Figure 9. The majority of the instructor's questions received a response within two seconds. Her students contributed greatly to this finding as a 57% of the questions were identified as *Immediate Student Response.* The instructor responded to 23% of the questions within two seconds. Wait time was used in numbers less than *No Wait Time* since *Minimal Wait-Time, Wait-Time,* and *Extended Wait-Time* together made up 20% of her questions.

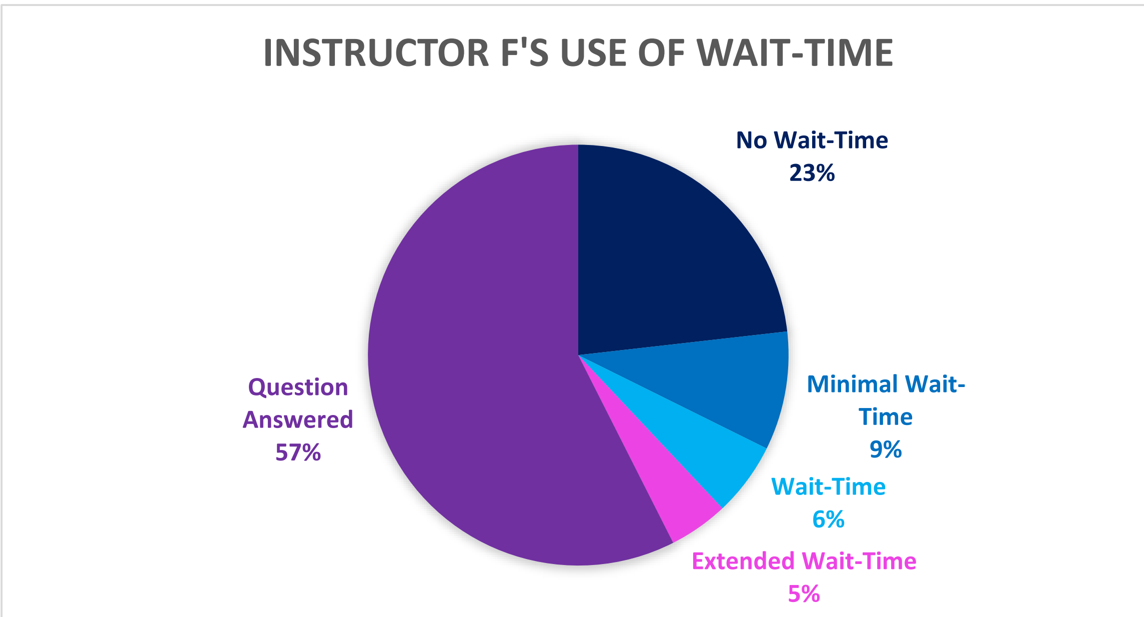

Figure 9: Instructor F's use of wait time

Figure 10, shows that each wait time category was observed daily in Instructor F's class during this study. The students responded within two seconds to at least of half of the questions on each day. Day 2 was striking in that the students responded within two seconds to around 70% of the instructor's questions. Further, less than 5% of the instructor's questions were followed by wait times over three seconds on that day. Questions were followed by *No Wait-Time* more often than *Minimal Wait-Time*, *Wait-Time*, and *Extended Wait-Time* collectively on each day with the exception of Day 3.

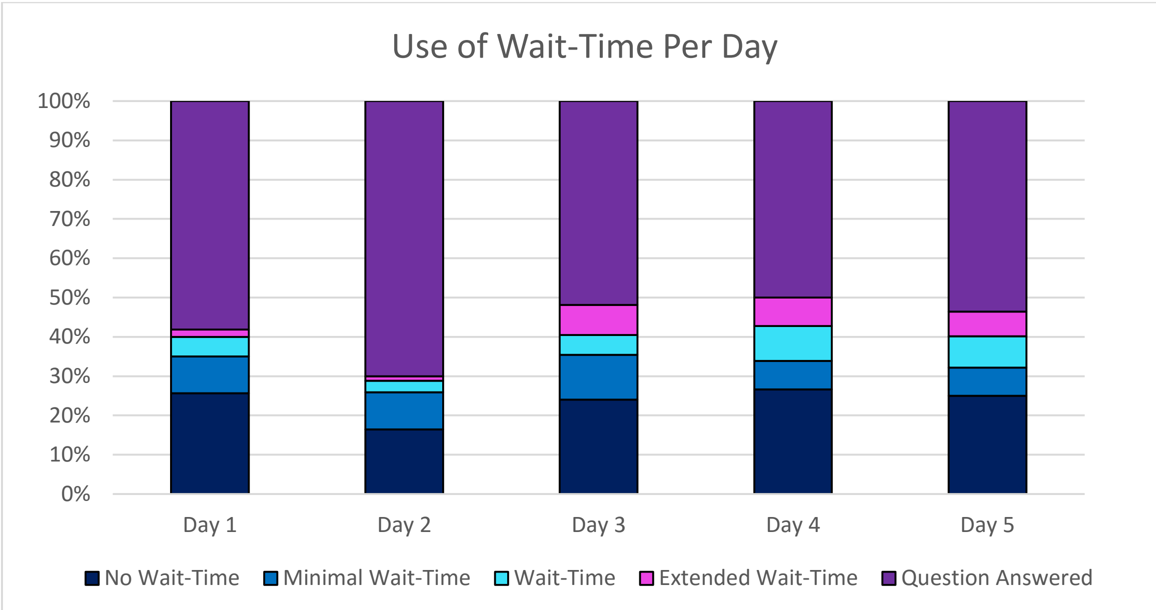

Figure 10: Instructor F's use of wait time per day

A look at wait time for each question type, shown in Table 18, indicates that Instructor F did not use a particular kind of wait time for specific question types. Six question types were coded as each wait time category, namely, *Gathering Information*, *Checking Understanding*, *Requesting Validity*, *Generating Discussion*, *Meaning/Relationships*, and *Probing*. On the other hand, the *Inserting Terminology*, *Extending Thinking*, and *Establishing Context* types were never followed by pauses of more than two seconds. The only category coded across all question types was *Immediate Student Response* indicating that students answered questions within 2 seconds despite the type. The most intriguing finding from Table 18 is that *Probing* was the only question type where wait time was provided more often than not.

Table 18: Instructor F's Use of Wait Time by Question Type

	No-Wait	Minimal-Wait	Wait-Time	Extended Wait-Time	Immediate Student Response	Total
Gathering/ Leading	109	47	24	19	322	521
Generating Discussion	15	4	3	2	10	34
Requesting Validity	10	5	1	4	12	32
Checking Understanding	17	6	6	3	9	41
Meaning Relationships	5	1	3	1	5	15
Linking Applying	2	1	1	0	4	8
Orienting/ Focusing	4	1	0	0	9	14
Inserting Terminology	0	0	0	0	5	5
Probing	4	1	3	4	36	48
Extending Thinking	0	0	0	0	3	3
Establishing Context	2	0	0	0	0	2
Total	168	66	41	33	415	723

Questioning without wait time. Questions identified as no wait time were followed by an instructor comment, explanation, question, or answer. Instructor F answered approximately 45% of her questions followed by no wait time as such this was the typical response to these questions. The questions she answered were primarily of the *Gathering Information* type, yet there was evidence of this occurring with the *Orienting/Focusing*, *Probing*, *Generating Discussion* and *Establishing Context* types. One example took place as they talked about curve sketching. She said, “So at x equals one; is the derivative positive, negative, or zero?” In less than two seconds she answered, “It’s zero.” Another example took place as she prepared to ask the students about the minimum and maximum of a piecewise function. Instructor F asked, “Where does f achieve, well first of all is this function continuous?” and responded “It looks it” in less than two seconds. Other examples of *Gathering Information* questions the instructor answered are “What functions have zero derivative?”, “Ok, now what is a of 50?”, “So now what is the derivative of s with respect to t?” and “So what does the law of similar triangles say?” All of these questions were questions that students should have been able to answer based on known information from either Instructor F’s class itself or previous courses.

Instructor F followed her questions with a comment or explanation mainly when the questions were of the *Gathering Information* and *Checking Understanding* types. Questions that *checked understanding* like “Does this make sense?” and “Questions?” went unanswered verbally, yet the students could have responded nonverbally within this time. On the other hand, *Gathering Information* questions followed by an explanation or comment were sometimes later answered by the instructor or a student. One example of this is in the excerpt below. It took place as they talked about the second derivative and curve sketching.

> I: Where is the second derivative zero? Well I need to know what the second derivative is. What is the second derivative?
>
> S: $6x$.
>
> I: $6x$. Ok this is zero. This is zero at x equals zero.

The instructor’s original question was immediately followed by a comment. She asked another question that a student answered before repeating their answer and answering her original question.

At times Instructor F asked a question, paused, less than two seconds and asked another question. This was observed for *Gathering Information* questions for the most part. Back to back questions such as “So what is another characteristic of the first derivative? It might be instead of positive, it could be?” were observed. A student answered the subsequent question, which was normally the case. In few instances there were more than two questions in a row. One took place as the instructor asked students about finding the derivative of the cosecant of t if they did not remember that is was “cosecant tangent.” She reminded them that cosecant could be written as one over sine and asked “What is the derivative of one over sine t in this way? [< 2 seconds] Like how can I, what’s the derivative when I have one over something? [<2 seconds] What does the chain rule say the derivative of that is?” [>2 seconds]. These questions went unanswered, however, the instructor later asked questions that led students through using the chain rule.

Questions followed by no wait time were generally answered by the instructor. Other questions were followed by an instructor comment, explanation, or question and at times went unanswered, but if they were answered it was most often by a student.

Questioning with wait time. As previously stated, multiple question types received wait time, which is recognized here as *Minimal Wait-Time, Wait-Time,* and *Extended Wait-Time* collectively. Roughly 58% of questions that had wait time were followed by a student answer whereas a low 6% were instructor answers. Like questions with no wait time, the instructor also responded with comments, explanations, and other questions. Questions were sometimes answered later or nonverbally.

All of the questions answered by Instructor F were of the *Gathering Information* type. For example she asked the students what else they needed to look at with the derivative to find critical points and paused between three and five seconds before answering that you would need to consider where it is undefined. The instructor was observed waiting more than five seconds at times before answering herself. The students answered questions of various types after some kind wait time. These questions were mainly *Gathering Information* questions and they were answered with wait times of two to three seconds in most cases.

A noticeable portion of the questions given wait time were followed by an instructor comment or explanation. Regardless of the question type these questions were rarely answered later in the class session, but many could have been answered nonverbally. A number of these questions were of the *Checking Understanding* and *Requesting Validity* types where answers as simple as "yes" or "no" would have sometimes sufficed like "This is the derivative of g with respect to s where s is a function of t. Right?" and "Ok, so questions about this?" A student may have only found it necessary to respond verbally if they had questions or wanted to express why they disagreed with something. On occasion the instructor's comment was followed by another question which was answered instead.

Rarely did Instructor F ask back to back questions when some kind of wait time was provided. She only did this nine times out of more than 100 questions. In a couple of instances she asked the same thing after getting no response. For example, she *probed* a student's thinking by asking "How did you do this?" She waited between three and five seconds and after no response said "It's correct; it's very good so explain for us please." The student immediately responded to the subsequent question. In other cases, Instructor F asked an alternative question before getting back to the original question. Consider the following passage, which took place while discussing a related rates problem.

> I: …Well I could have said you know what the radius is changing at a rate of three inches per second. Ok, what does that mean r is as a function of time? [>5 seconds] So after one second where is the radius going to be?
>
> S: Three.
>
> I: After two seconds?
>
> S: Six.
>
> I: Six. So what's $r(t)$?
>
> S: $3t$.
>
> I: $3t$ right?

The instructor provided *Extended Wait-Time* before asking another question in this passage. After asking two questions about what the radius would be after a specified number of seconds she reworded her initial question and got an immediate student answer.

Questions identified as *Minimal Wait-Time, Wait-Time,* and *Extended Wait-Time* did not represent a large portion of the questions asked, but these moments gave students time to think at the least. The students often answered these questions, and the questions answered by the instructor were all based on recall. Instructor F's most common response was an explanation or comment behind questions that were either answered later or nonverbally.

Summary. A pause of more than two seconds after a question was hardly observed in Instructor F's class. This is mostly attributed to the fact that the students were quick to answer more than half of the questions posed. Aside from that, Instructor F was more likely to offer a response of her own within two seconds than to wait on a student response. The instructor's typical response was an answer when wait time was not provided, but when she waited the students often answered. Interestingly, she answered her own questions less when she provided wait time. Wait time was observed for numerous question types although it was not observed in abundance.

Classroom Setting

The questions in Instructor F's observed classes were asked while engaged two in-class activities. Lecture-discussion was the dominant in-class activity. There was some time spent in whole group discussion as well, though it was minimal. Instructor F stood at the front of the room and lectured daily from notes that she brought to class. Each day she lectured on new material, however, on three days she reviewed for a small portion of the class. The majority of her questions were asked as she lectured on new material as this is how the bulk of the class time was spent.

The students generally answered the instructor's questions immediately during lecture-discussion regardless of if it was review or new material. Further, they answered more questions than the instructor answered herself during this in-class activity. When more wait time was provided, they also tended to answer the questions. The instructor's questions were followed by wait time more often than not when she reviewed, but the opposite held true when she introduced new material. All 11 question types were present when Instructor F held lecture-discussion on new material, but only the *Gathering Information*, *Probing*, *Terminology,* and *Checking Understanding* questions showed up during review. The later three types together made up 12% of the review questions.

Whole group discussion took place on Days 2 and 3 after taking a quiz. There was not much time devoted to this in-class activity. It appeared the purpose was to simply go over the answers to the quiz. Eight questions were posed during the whole group discussions across both days. All of the questions posed with the exception of one *Probing* question were of the *Gathering Information* type. Each question received a student answer in less than 2 seconds, so there were no opportunities for the instructor to offer wait time.

Whole group discussion was a miniscule part of Instructor F's class compared to lecture-discussion as such there is not much a comparison. The questions posed during both in-class activities were similar in that they were dominated by *Gathering Information*. Further, it can be concluded that the students were engaged in a call and response fashion during both. Bigger differences were seen within the lecture-discussions in terms of new material versus review.

Summary

The primary function of the questions observed in Instructor F's class during this study was to *Gather Information.* Most questions could be answered based on established facts or procedures and generated short responses. Instructor F was observed answering her own questions, commenting/explaining, and asking back-to-back to questions. She asked more than 700 questions, and all 11 question types were present indicating that the students were indeed given some opportunities to engage in mathematics beyond recall.

It was typical for the students to answer questions in two seconds or less regardless of the question type. They also answered many of the questions when wait time was provided. The instructor too answered questions within two seconds and when she provided wait time; and did so during the former more than the latter. Wait time was provided for about 20% of the questions and it was utilized with most of the question types. It was generally used at a frequency less than no wait time, however, it was observed with a frequency higher than no wait time with the *Probing* type.

Two in-class activities, lecture-discussion and whole group discussion, were put to use in Instructor F's class sessions. All wait time categories were observed during lecture, yet there were differences within how it was used with questions about new material versus questions for review. The instructor paused for wait time more often than not when they reviewed. The reverse held true for new material. She asked questions from all 11 question types when the focus was new material, but fewer types were used during review. Only two question types, *Gathering Information* and *Probing* were asked during whole group discussion. Wait time was never provided during this in-class activity because the students answered each question within two seconds.

Case of Instructor T

Instructor T's Calculus I class took place in a computer lab with three rows of computer tables that seated pairs. There were five unconnected computer tables in any given row and each one held two computers with four computer screens. The entire front wall functioned as a dry-erase board, however, the instructor spent little time writing on it. A projection system used to occasionally demonstrate something on Mathematica was front and center. Between 25 and 28 students attended class during this study where they were engaged in student-student discourse daily. The students did most of the talking which can be attributed in part to the class structure.

A day in Instructor T's class began promptly with a warm-up that was either a single problem out of the electronic textbook, or a problem written on the board by the instructor. The students were encouraged to think about the warm-up individually first, but they were expected to collaborate with each other after doing so. It was clear that this was an expectation because the instructor was observed saying things like, "Are we collaborating here?" and "Alright let's hear some talking. Communicate with each other." Warm-ups gave students an opportunity to engage with each other, however, the instructor regularly walked around the classroom and checked in with students.

The warm-up problems were discussed as a whole group, and they included numerous instructor-student interactions that were shaped by the instructor's questions. Students were assigned a "challenge problem" at the conclusion of the warm-up on all observed days except for one. On this particular day the students worked on a homework problem from the previous class session and discussed it before working on the challenge problem. In groups of six or seven, students worked on one challenge problem found in the electronic textbook or one provided by the instructor on the board. Two days the entire class worked on the same problem in small groups, and three days the small groups worked on different problems. For example, on Day 1 each group had to find the asymptotes of a function, but they had different functions. Day 2 everyone worked on a problem from the book that said, "If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box." The challenge problem was discussed as a whole group when the class worked on the same task. Student representatives from each group presented their work when they explored different problems, but this too resulted in what was like a whole group discussion.

Students in Instructor T's class experienced Calculus I in an environment where the instructor's talk occurred in brief spurts intertwined with their own during discussions about mathematics. The instructor did not stand at the front of the room lecturing about how to do problems, but he did occasionally spend time explaining things to students and adding on to their explanations. Most of the instructor's talk turns were well under 40 words. His longest two talk turns were a little less than 500 words and both included directives as students prepared to work on their challenge problem. He asked students an ample amount of questions while they worked on a variety of topics. An overview of the observed days can be found in Table 19.

Table 19: Overview of Instructor T's Class by Day

Day of Observation	Overview of Class Session	Classroom Context
Day 1	• Warm-up (Using graphs of first and second derivatives to answer questions about function) • Challenge Problems (Finding asymptotes of functions)	• Small group work • Whole group discussion • Small group work • Student presentations
Day 2	• Warm-up (Maximize the area) • Challenge Problem (Optimization)	• Small group work • Whole group discussion • Small group work

Table 19 Continued

	• Began another problem (Optimization)	• Whole group discussion • Individual work (finish for homework)
Day 3	• Warm-up (Distance between two points on the coordinate plane) • Homework problem from Day 2 (Optimization) • Challenge Problems (Optimization)	• Small group work • Whole group discussion • Small group work • Whole group discussion • Small group work
Day 4	• Warm-up (Introduction to idea on antidifferentiation) • Challenge Problems Continued from Day 3 (Optimization)	• Small group work • Whole group discussion • Small group work • Student presentations
Day 5	• Warm-up (Antiderivative Problem) • Challenge Problem (Estimating distance traveled over time period given a table)	• Small group work • Whole group discussion • Small group work • Whole group discussion

Questions by Type

Instructor T's questions comprised nine types as seen in Table 20. *Gathering Information* questions occurred at the highest frequency (around 32%). He also used *Requesting Validity* and *Checking Understanding* questions in high numbers. Further, students were engaged with questions that asked them to think deeper with the *Probing, Generating Discussion*, and *Meaning/Relationships* types. Other question types were observed minimally, but still contributed to Instructor T's overall questioning. There was no evidence of the *Establishing Context* or *Linking/Apply* question types, as such they will not be discussed my analysis.

Table 20: Question Types Used by Instructor T

Question Types	Frequency (Percentage)
1. Gathering Information	106 (31.7%)
2. Inserting Terminology	5 (1.5%)
3. Meaning/Relationships	15 (4.5%)
4. Probing	37 (11.1%)
5. Generating Discussion	33 (9.9%)
6. Extending Thinking	2 (0.6%)
7. Orienting/Focusing	2 (0.6%)
8. Checking Understanding	60 (18.0%)
9. Requesting validity	74 (22.2%)
Total:	334 (100.1%)

Figure 11 displays the percent of questions identified as each question type by day. It was observed that six or more question types were utilized on any given day. Additionally, there were six question types common to each day, namely, *Gathering Information, Generating Discussion, Checking Understanding, Requesting Validity, Probing,* and *Meaning/Relationships.* This section focuses on the findings gathered within the question types common to each day, but also gives attention to questions types with minimal occurrences in a single subsection.

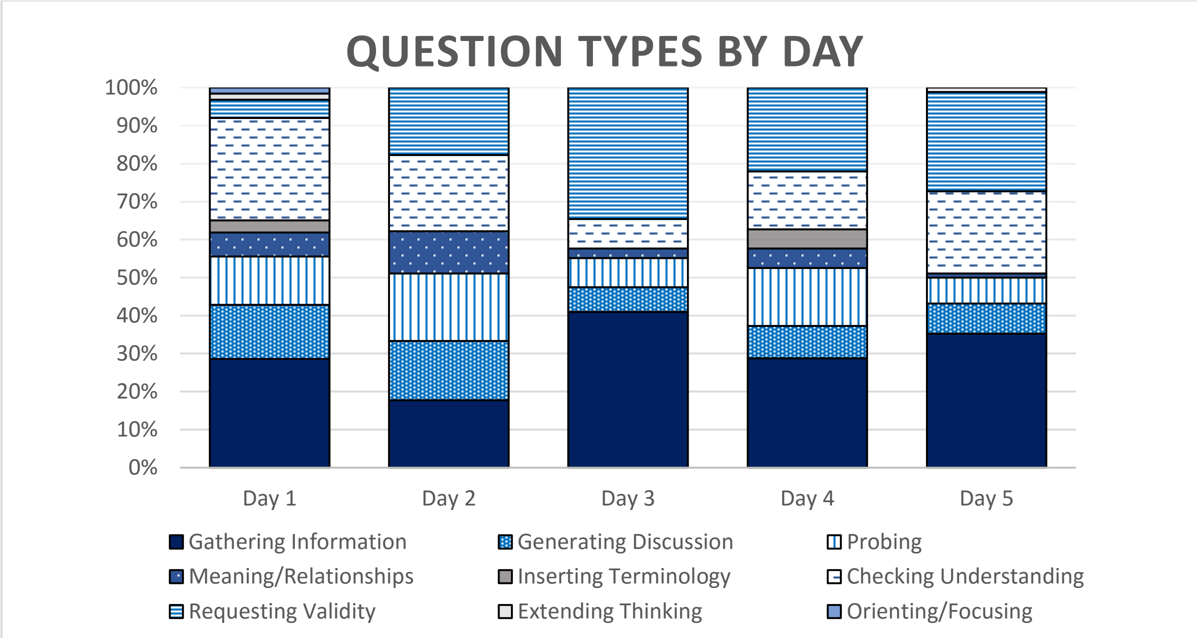

Figure 11: Instructor T's question types per day

Gathering information. Instructor T gathered an array of information that can be described somewhat differently. It was observed that the word 'what' usually shows up somewhere in these questions, but this was not always the case. Consider the excerpt below where the Instructor T asks multiple *Gathering Information* questions. The excerpt comes from a discussion that was generated after a student presentation on the asymptotes of a function. It begins with a contribution offered by another student in the class.

> S: If the degrees of the denominator and the numerator are equal, so like x squared and x squared or x cubed and x cubed, so the degrees are equal. You find the horizontal asymptote by just taking the ratio. So one to one so horizontal asymptote at y equals one.
>
> I: Ok. I'm going to revoice you kind of in a question. What is the coefficient of the highest degree term in the numerator?

S: One.

I: That's right. One. What is the coefficient of the highest degree term in the denominator?

S: One.

I: That's right. One. What's one divided by one?

S: One.

I: Where do we have our horizontal asymptote? y equals one right? So that's how we analytically find the horizontal asymptote at y equals one.

Four *Gathering Information* questions can be pulled from this selection. The first two, "What is the coefficient of the highest degree term in the numerator?" and "What is the coefficient of the highest degree term in the denominator?" ask that students identify something based on known information. The third question asks students to execute a computation; specifically, "What is one divided by one?" The final question simply asks students to state a previously discussed answer. Instructor T's purpose for asking these questions was explicit in his final statement.

Other questions understood as this type asked students to recall information that they have previously learned. For example, "What do I know the derivative is equal to at this peak?" and "Before we shift enter here, by raise of hand can someone tell us what we should expect the graph of this to look like?" The instructor asked the first question about the graph of a quadratic function which students know has a derivative of zero at its maximum by this point in the class. The second question was in reference to the derivative of that particular quadratic. Finding derivatives is common knowledge by this time meaning the students know that taking the derivative of a quadratic produces a line. These particular questions were asked while the instructor unbeknownst to the students at that moment used Mathematica as a means of showing them a way to confirm their findings for optimization problems.

Instructor T also asked students questions that involved stating steps of a known process. In their transition from a warm-up on optimization to a discussion on an optimization homework problem, the instructor asked students questions like, “What’s step two in this process?”, “What is the fourth step that we gave in the steps to find or optimize?”, “Was there something about introducing appropriate notation? Did I skip that?”, and “What’s the last step in conducting optimization problems?” The students were not asked to state these steps back to back; they were identified before discussing each step in the problem of concern. In one instance the instructor asked students to state a known formula. This was the distance formula, which the students needed in order to complete their task of finding the point on a graph closest to the point (3, 0).

Multiple students were asked what they got as an estimate, during group work, on a problem where their estimate was actually the answer to the task. The instructor used this information later when he connected the methods students applied to get their estimate to the ideas of over and under estimation and Riemann Sums. Other questions were asked for the instructor’s understanding of students’ written work during group work. He asked one student a question about a figure she had drawn, “Is this the refinery here?” He asked another, “Do you have an equation where h is isolated?” Questions that *Gathered Information* usually yielded short responses that were either correct or incorrect. At times, Instructor T used his questions to request the validity of these responses.

Requesting validity. Statements followed by “Right?” were regular incidents in Instructor T’s observed classes. They usually showed up after he expressed ideas or revoiced students’ responses, but “Right?” sometimes occurred in the instructor’s responses to students’ questions. They served the same purpose regardless of where they took place and that was to get students to validate others’ claims. The students often responded to these questions, yet the times they did not still served as opportunities students had to consider the soundness of the claims made. The verbal responses in agreement to these questions normally included “yes,” or “right” as one may suspect, however, there were a few instances where “Right?” sparked a student question. This section paints a picture of how “Right?” was typically used in Instructor T’s Calculus I class.

During the whole group discussion of a warm-up that directed students to find the antiderivative of a given function, the instructor asked a student how he would check their answer. The student answered, “Take the derivative.” Instructor T’s responded

> Ok, just take the derivative of whatever you think the antiderivative should be. Right? So just for thoroughness sake I’m going to go ahead and do that $f'(x)$ equals. Oh four times three is 12, four minus one is three. These are all applications of the power rule of differentiation, and it does check out. Right? So although these might seem like somewhat intimidating questions at first…

Notice that within this response there are two statements followed by “Right?” and neither received a verbal reply from students. The first one occurred after the instructor revoiced the student’s answer and asks students to validate that answer. The second was after the instructor found the derivative on the board and asks students to confirm his claim that their answer “checks out.”

The following example illustrates moments that students validated the instructor’s claims. Instructor T held a discussion with a small group of students as they worked on a problem that asked them to find the dimensions of a can that would minimize the metal used to make it. The instructor asked the group for their surface area equation considering the fact that the can does not have a lid. The students gave their response, and the subsequent dialogue transpired.

> I: It’s just pi r squared because there is only a bottom. There’s no lid.
>
> S: Oh ok.
>
> I: And I thinks that’s going to be your optimization equation. Right?
>
> S: Yes. Then we want to minimize or maximize.
>
> I: Look at the problem. I think you guys want to minimize the amount of material used. Right?
>
> S: Yea.

The equation that the student stated was for a can with a lid, so the instructor corrected him. The student responded and fixed the equation on his paper. The instructor then used “Right?” to ask the students in the group to confirm his assertion that the equation they now have will be their optimization equation. A group member agreed that this would be the case and offered a next step, which included two alternatives. The instructor gave a directive followed by a request for students to validate his recollection of what the problem was asking them to.

There was one other question that was identified in this category that acts the same way. It was the statement "So it looks like the average of those two velocities will be 46.5 times 5 and then we have one more time interval" followed by "Don't we?" This too asked students to validate a claim. The students had to decide if they agreed with something with questions of this type, yet other questions asked that they determine if they understood something.

Checking understanding. Students in Instructor T's Calculus I class worked in groups regularly. He often walked around shortly after the problems were established and asked a similar question each time that required the students check their understanding of the problem. This included questions like "This make sense?" and "Make sense?" These exact questions were posed while students worked on a warm-up that asked them, "what function, $f(x)$, would produce the derivative $f'(x)$ =$12x^3 + 6x - 4$?" During this particular warm-up, questions of this essence were met with a simplistic, "Yea" and the discussions ended.

The students' responses did not always indicate that the task made sense, which led to a little more dialogue, but these discussions were generally short. Questions that were understood as this nature often looked a lot different than "Make sense?" Consider an excerpt from a similar warm-up which asked students to, "Find the antiderivative of $f'(x) = \frac{1}{2\sqrt{x}}$."

I: Linda, Rickie. How we doing?

S: Not that great.

I: Not that great. Think about what's happening with exponents here. Like if you rewrote this with a rational exponent. That might help you think about where this comes from…

Here the instructor asked a pair of students how they were doing, which was taken as asking if they understood the problem. The student's response resulted in a hint from the instructor.

Instructor T also walked through checking in with groups asking them how they were doing or if they had any questions as they worked on their challenge problem(s). Interestingly, *Checking Understanding* was displayed by simply calling students' names as he made his way through the classroom. Consider the following excerpt, from Day 5, as an example.

I: Steven, Dee, Twan.

S: I have a general idea of how to do it. It's the area underneath the curve.

S: Yea.

I: Ok well you have a little bit of a picture right?

S: Yea.

I: So just come up with a way to estimate the area. You're right that you want to find what you said. There are some good ways and some not so great ways to do that…

It can be gathered from the student's response that the instructor calling their names was asking them to reflect on their understanding of the problem.

At times students were challenged to check their understanding of explanations only. This took place during all aspects of the class, and considered the explanation(s) of the instructor as well as other students. He asked questions like "Does everyone hear what he is saying?" and "Did everyone catch the logic?" in relation to students' explanations. Similar questions were posed when he asked students to check their understanding of the explanations he offered. The most common was some form of "Do you know what I mean?"

Questions that asked students to check their understanding of problems and/or explanations resulted in discussions of varying lengths and sometimes yielded no student response at all. These are moments students had to think about their understanding despite the responses provided. A simple "yes" is often response enough for these questions, therefore, nonverbal gestures may have been acknowledged. While this was true for questions that *Checked Understanding*, it was not true for questions that aimed to get students engaged in the classroom discourse.

Generating discussion. As previously stated, the students participated in whole group discussions on each observed day. Instructor T used *Generating Discussion* questions in two distinct ways to get students talking. Some of these questions asked students to share their thoughts and ideas with the class. Additional questions sought to get contributions from other students in the class.

Whole group discussions of problems often began with the instructor generating discussion with questions that asked the students to share their thoughts or ideas with the class. The whole group discussion on the warm-up from Day 2, for example, began with "Now, let's get someone to start us off. Looking at my diagram what might you add to or did you add to it to help you model or tackle this problem? What do we do? Cornell." In a similar fashion, Day 5's warm-up discussion was initiated with the instructor saying, "…What insight did you or people at your table have on tackling this antiderivative question? Rhonda I see your hand." Whole group discussions on a single problem that students worked on in groups were generated the same way. Consider the following exchange:

I: Ok Evette's going to go first. I'm going to call this, I'll put the work over here, Evette's method. Evette you might not have expected to be a famous mathematician when you came in today.

S: We can call it Evette's Theorem.

I: Ok let's go on to the next volunteer. Just kidding. Evette give us your method aka Evette's Theorem.

S: So what I did was I looked at the difference between the intervals on the table. So for instance between zero and five I found a number in between from 25 and 31.

Similarly, when students worked on different tasks the discussions commenced with group representatives being called to the board to share their take on a particular problem. These questions prompted students to share their thoughts with the class, and at times the instructor invited other students to add to the discussion.

On any given day Instructor T could be observed asking students to add to whole group discussions on their warm-up or challenge problem(s). These questions were not asked often in comparison to others in the generating discussion category. For example Days 3 and 4 had exactly one instance of this kind of request, namely "Someone else, what do you use or how do you think about this?" and "Anything else? Catina." The other days did not have many more, but Instructor T always got a response from students when he invited them to contribute to the discussion. These questions were posed in two ways; asking a particular student for his/her contributions and asking for a volunteer from the group to add to the discussion. The examples provided below show the instructor's conscious efforts to get multiple students engaged in discourse.

Instructor T generated discussion by requesting contributions from other students in the class on Day 1 after a group's representatives presented their graph of a function, its asymptotes, and their approach to finding them. These requests were made generally with "Some other contributions?" and specifically with questions like "Winston what do you got to add?" On Day 2 instead of inviting students to add to a discussion about a problem that they had not completed themselves, he engaged different students in a discussion that resulted in completing the warm-up they had all explored. This is demonstrated by "Let's get by raise of hand someone to give us kind of like the next big step?" and "Now what? Does anyone have a good next step for us?", for example.

A short excerpt can be used to sum up how Instructor I used his questions to get contributions from members in the class on Day 5. The following exchange occurred during their whole group discussion on their challenge problem.

> I: …So visually Mook is estimating the area under this curve that would connect these dots with this blue rectangle. Do you see how that rectangle initially overestimates things, but then it underestimates things? Maybe then providing an overall decent estimate. By raise of hand, can I get someone else to share their method with us? What other methods are out there? Walter.
>
> S: Mine is a lot like Keith's, but I pretty much just added up all the velocities, like from 0 to 5 there's a difference of 6.

It can be gathered that multiple students were asked to share their method to completing the same task during this particular whole group discussion.

Although questions that asked for contributions from other students in the class were not observed in numbers, they are indication that it was important to Instructor T that multiple students engaged in discussions. *Generating Discussion* questions worked for Instructor T, in terms of their purpose, because without a doubt students responded and the discussions went beyond the requested contribution(s).

Probing. Like *Generating Discussion* questions, *Probing* questions requested contributions from students, however, in this case they asked students to further express their thinking. Questions that probed into students' thinking did not constitute a large percentage of Instructor's T questions, yet those he asked point to his use of various kinds of *Probing*. These questions primarily occurred during whole group discussions and asked students to elaborate in some capacity. The instructor's use of questions involving "why" were the most prevalent in his probing.

In one instance a student shared his method to finding an estimate of the total distance traveled during the first 30 seconds based on a table of rates and corresponding times. The instructor asked for the units, the student responded "feet," then the instructor said "Feet. Ok so one thing that's kind of strange is that we have an area under a curve with units of feet not feet squared. Why does that happen?" In essence, Instructor T was asking the student to come up with a reason as to why something was the case. Another time, the instructor asked a student why they did something. This took place during an optimization problem involving area given a particular amount of material. A student expressed that the equation $2x + y = 2400$ could be reconfigured and written as $y = 2400 - 2x$. The following exchange transpired.

I: Alright, so Tisa does an algebraic manipulation on this equation that tells us something about how much fence we can use. Right? Everyone see that? So that's the what. Tisa why did you do that?

S: Um just so it's kind of like easier cause you know that area is x times y.

Probing questions containing "How" asked students how they did something or how they knew something. For example, "Can you explain how you got the 37.8 ft/sec?" Here the question *Probed* a student about their approach. It is important to note that questions that ask students about their approach may not ask "how" but "what?" For example, "Nathan do you want to tell us what you did next or should I go to someone else?" These questions differ from those that ask students how they know something because they exceed students' knowledge about how they solved a problem or how they found a particular number or answer, and ask students to back-up their claims. In a problem that connected a function to its first and second derivative, a student claimed that the concavity of a function changed at $x = 2$. Instructor T followed that up with a question that restated the claim and asked "…how do we know?"

The instructor asked two questions that *Probed* for meaning in that they asked a student what something meant in the context of a problem. A pair of students were asked to articulate what their answer meant in terms of the optimization problem they were presenting and the following discourse transpired.

I: But like what does it represent exactly?

S: The area of that half or that semicircle.

I: Now you said this is a perimeter.

S: Uh yea.

I: So what does it really represent? Notice you have x, two h's. (Motions a semicircle)

S: The perimeter of a semicircle.

Probing questions generally asked students "how" and "why" giving them opportunities to think about and discuss mathematics beyond the surface. Similarly, questions of the *Meaning/Relationships* type gave students a chance to engage in learning mathematics more deeply.

Meaning/Relationships. Instructor T used questions in this category sparsely, yet they represent times students had to make meaning of concepts. These questions asked students to make links between or identify relationships. During a warm-up the students were given a graph and were asked to answer questions based on if the graph was the function f, the derivative of f, or the second derivative of f. This allowed them to make links between mathematical ideas and their representations. For example, the students were asked "By raise of hands can someone help us out with locating x value wise the points of inflection on this graph, assuming this is the graph of f?" and "Notice between zero and one f prime is negative. By raise of hand what does it tell us about the slopes of the tangent line Jenny to graph of f?"

Another example took place as a pair of students presented their work finding the dimensions of a Norman Window that would produce the largest surface area. Instructor T paused one of the students while she explained their work saying, "I'm going to pause you just for a second. I think other people may be wondering this as well. You have this term pi x

over two. Where does that show up in your picture?" Here the student was being asked to link the term in their algebraic work to the figure they had drawn. This question not only allowed the individual to make meaning, but it clarified what the instructor saw as the potential confusion of others. The instructor probed the student for meaning after their response.

In one lesson Instructor T said to the class, "This is going to be kind of strange, but what are the units of this area?" The instructor had previously asked the students if they maintained a velocity of 60 m/h for two hours how far had they gone. A student responded "120 miles." The instructor drew a graph on the board, and then asked about the area of the rectangle. This time a student responded "120," so the instructor asked about the units of the area. This question was identified as making meaning because it challenged students' traditional understanding of area as having a squared unit to see if they could make meaning of the area beneath the velocity curve being miles.

The students answered these questions whenever they were asked, suggesting that they were comfortable when the instructor sought to have them make meaning. These questions represent important moments in Instructor T's questioning because they support conceptual understanding. *Meaning/Relationship* questions did not emerge regularly, but they showed up at least three times as often as *Inserting Terminology, Extending Thinking,* and *Orienting/Focusing* questions.

Additional question types. There was minimal evidence of questions that did not fit into the discussed question types. This included five questions that focused on students' use of terminology, two that oriented and focused their attention on the problem, and two that extended their thinking. An example of each question type can be found in Table 21.

Table 21: Examples from Instructor T's Low Frequency Question Types

Question Type	Frequency	Example
Inserting Terminology	5	The what, on the top?
Orienting/Focusing	2	Alright so what's unique about this cylinder?
Extending Thinking	2	Can we find the actual average velocity for all seven velocities? Can we use that here?

The *Inserting Terminology* example took place as a student shared how his group found the oblique asymptote of a function. He said "…I guess when I look at it, the way you figure out its y equals x is because the coefficient on the top is one greater than the one on the bottom." Instructor T asked, "The what, on the top?" so that the student would use the appropriate term which he responded was "the degree." The *Orienting/Focusing* question was used to focus a group of students' attention on something that was important to recognize in terms of solving a particular optimization problem. The students' thinking was *Extended* while discussing different students' approach to finding the distance traveled given velocities and times. He asked the example question after a student explained how he only used six of the velocities to find the distance.

Summary. Instructor T's questions functioned many ways during his observed Calculus I sessions. Questions that *Gathered Information* had the highest frequency (around 30%), yet they were not used excessively. His students were given many opportunities to engage in discussions about mathematics that exceeded answers with *Generating Discussion, Probing*, and *Meaning/Relationships* questions. The students generally responded to the instructor's questions despite the different types and they were given a range of time to do so.

Wait Time

Instructor T displayed a range of wait time use during his observed Calculus I class sessions. There was evidence of all coded wait time categories meaning his students were not limited to having less than two seconds to respond to his questions. Figure 12 provides the percents for each wait time category observed in his questioning. It can be extracted from the figure that more than half of the questions posed by the instructor were answered immediately by students. More often than not, Instructor T provided some type of wait time for students regardless of if it was *Minimal Wait-Time, Wait-Time,* or *Extended Wait-Time*.

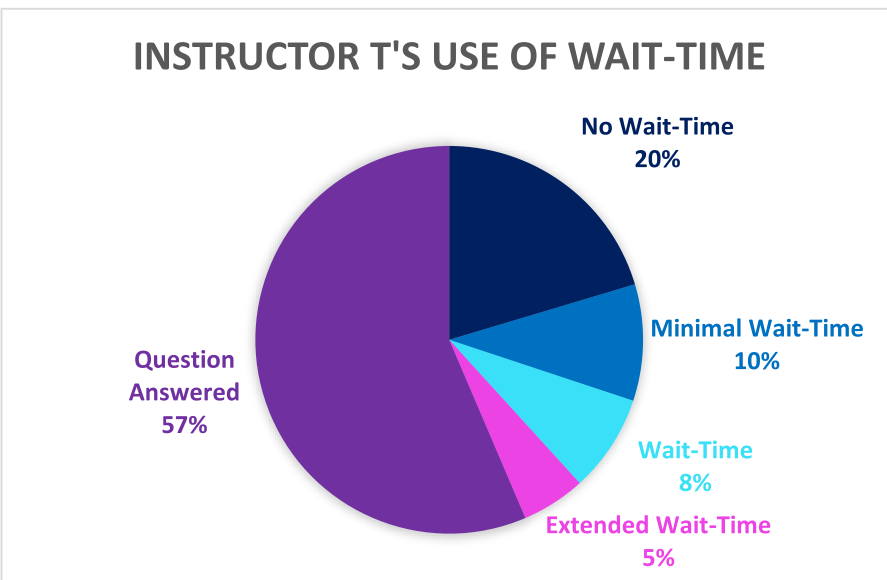

Figure 12: Instructor T's use of wait time

I found with a closer look at the wait time categories that it was commonplace for students to respond to questions promptly on a daily basis. I also found evidence of Instructor

T providing *Minimal Wait-Time, Wait-Time, Extended Wait-Time*, and *No Wait-Time* on each day. Questions were also given wait time more often than not on Days 4 and 5. Figure 13 supports these findings showing the percent of questions coded as each wait time category per day. These findings offer interesting information about Instructor T's broad use of wait time, yet looking at wait time per question type adds to understanding how wait time was used.

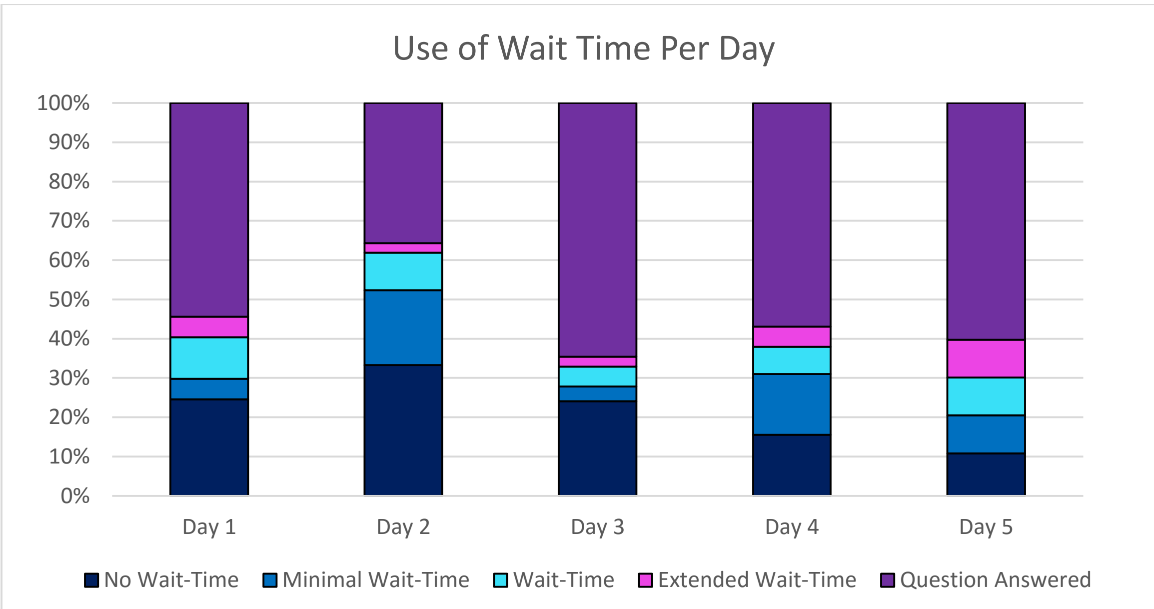

Figure 13: Instructor T's use of wait time per day

There was not a specific kind of wait time utilized for a particular type of question in Instructor T's class. Each category of wait time was used for *Gathering Information, Requesting Validity*, *Meaning/Relationships*, and *Checking Understanding* questions. The students responded to questions immediately for each observed question type. It was also the most common wait time category coded daily. Table 22 provides the frequency that each question type was coded as *No Wait-Time, Minimal Wait-Time, Wait-Time, Extended Wait-Time,* and *Immediate Student Response.*

Table 22: Instructor T's Use of Wait Time by Question Type

	No-Wait	Minimal-Wait	Wait-Time	Extended Wait-Time	Immediate Student Response	Total
Gathering/ Leading	23	12	7	3	58	106
Generating Discussion	11	0	3	3	12	33
Requesting Validity	1	8	9	4	49	74
Checking Understanding	18	6	5	1	26	60
Meaning/ Relationships	3	2	2	3	5	15
Orienting/ Focusing	1	0	0	0	1	2
Inserting Terminology	2	0	0	0	3	5
Probing	5	3	0	3	25	37
Extending Thinking	1	0	0	0	1	2
Total	65	31	26	17	180	334

Questioning without wait time. Questions followed by no wait time were identified in every question type used by Instructor T, however, they were mainly of the *Gathering Information, Checking Understanding,* and *Generating Discussion* types. It was rare that these questions were followed by the instructor's answer to the question. In fact, the instructor only answered 12% of these questions. The vast majority of the instructor answered questions were coded as *Gathering Information,* as such the instructor answering them did not lead to missed opportunities for the students to engage deeply. For example, the instructor answered his own question while discussing the solving of an equation. He said,

"…You could just multiply both sides by the entire denominator, and then what do you get? Two x minus five equals zero and we solve for x, you get five halves…" This is a question that students should have easily been able to answer based on their enrollment in a calculus course.

Instructor T often followed questions given no wait time with a comment or explanation and occasionally a question. Many of these questions were similar in nature to "Everyone catch that?" and "Questions?" and were identified as *Checking Understanding.* Students may have responded to these questions nonverbally during this time, or their silence could have been indicative of their understanding. Had students responded verbally to these questions, there would have been more evidence of questions with an immediate student response.

It is important to note that the instructor being the first to speak after he asked a question did mean that the questions went unanswered by the students. Approximately 26% of questions coded *No Wait-time* were followed by the instructor calling on a student to respond. In the following excerpt he asked a *Probing* question, paused for less than two seconds, and called on Talia.

> I: We're putting it on its own plot, but it should come down from left to right. How do we know that? Talia.
>
> S: Cause it has a negative slope.

In many cases students could be seen raising their hands, suggesting that they were prepared to answer these questions. Other questions were answered by students after the instructor finished a subsequent comment or explanation. The next example shows Instructor T adding a comment before calling on a student to respond to his question.

I: What's step two in this process? And folks please go step by step when you're doing your group work if you don't you're going to get stuck…Allison.

S: Draw a diagram.

As previously noted in Figure 12, a relatively low 20% of Instructor T's questions were followed by no wait time. The questions identified as *No Wait-Time* were unique in that a noticeable portion of them were in fact answered by the students. Further, many of the questions that were given no wait time could have been answered nonverbally by students.

Questioning with wait time. In this section the *Minimal Wait-Time, Wait-Time,* and *Extended Wait-Time* categories will be discussed collectively indicating that some form of wait time beyond two seconds was provided. Similar to questions coded *No Wait-Time*, the talk ensuing these questions was often an instructor comment or explanation. These situations are different in that questions coded with some form of wait time allowed students time to think before the instructor began to talk again. In the following exchange the instructor asked a question and paused between three and five seconds before making a comment.

I: …We talked about why you need that plus c during last class. It will come up again. For right now though, what questions or clarifications might you have? Be sure you can do this on your own. Nova what you got?

S: Would you ever be given c?

The instructor generally paused and commented after statements followed by "Right?" There was only one that received a verbal response from a student. This particular "Right?" was followed with a student question. These questions gave students an opportunity to determine if they agreed or disagreed with a claim regardless of it they got a verbal answer from students.

A student answer seldom followed questions identified as having some kind of wait time, yet similar to questions without wait time the instructor's questions were sometimes later answered by students. At one point Instructor T asked, "Why do I keep coming back to this 2.5 seconds?" He paused more than five seconds before calling on a student named Ralph to answer this question. Four distinct comments following a question caught my attention because they showed the instructor's cognizance of wait time as well as his desire for students to engage in answering his questions. Three of these comments can be found with the question they followed as well as the actual wait time provided before the comment was made in Table 23. The fourth was similar to the first one and is not provided. The questions found in the table also stand as examples of questions that were answered by students after an instructor comment. It is worth noting that he did not answer any of the questions that he asked that were followed by wait time.

Table 23: Comments Indicating Instructor T's Cognizance of Wait Time

Question with Comment	Wait Time
I: …Does anyone have a good next step for us? **Oh I'm getting my stretching in.** Post lunch stretch. Jenny.	6.42
I: …What happens to the concavity on the graph of f, at x equals two? And why? **Giving you a little think time.** Jenny nice and loud.	8.23
I:...Why does the area get unit feet instead of feet squared? **Just giving you some think time here.** Libby.	6.90

Summary. The majority of the questions in Instructor T's observed Calculus I sessions received a response within two seconds. More than half of the time it was a student answer. The responses provided by the instructor were rarely answers to his questions. In fact, he only answered his own questions when there was no wait time provided. The instructor's responses to his questions with no wait time rarely interfered with the students' ability to answer questions because they were often given a chance to answer the questions themselves either by acknowledgement or after a comment or explanation. Interestingly, the students answered questions more often in less than two seconds versus when wait time was provided. Wait time gave students time to think if nothing else and Instructor T was obviously cognizant of it.

Classroom Setting

Instructor T's classes were made up of four distinct activity types, small group work, whole group discussion, student presentations, and individual work. The first three occurred regularly. The students worked in groups daily on both a warm-up and a challenge problem, yet there were differences in both the frequency and type of questions posed during these activities. First, the instructor posed questions daily while students worked on the challenge problem. Second, the instructor asked minimal questions as students worked on the warm-up and all of them were identified as *Checking Understanding*. Instructor T asked 100 more questions (from multiple types) as the students engaged in working on challenge problems than he did when they were working on warm-up problems. In addition to *Checking Understanding*, there was evidence of *Gathering Information*, *Requesting Validity,* and *Probing.* There was also four *Orienting/Focusing* and *Generating Discussion* questions. Questions posed during the warm-up were either answered immediately by students or followed by *No Wait-Time,* yet questions during the challenge problem were sometimes given wait time.

Similar to group work, whole group discussions took place daily and encompassed a variety a question types. All of the question types observed during group work were present during this in-class activity in addition to *Meaning/Relationships, Extending Thinking,* and *Inserting Terminology.* Instructor T asked most of his questions during whole group discussions and also provided wait time more often here than in any other in-class activity.

Student presentations were a lot like whole group discussions in that multiple question types were utilized. Most of the questions types mentioned previously showed up during student presentations. Wait time and no wait time were used similarly in terms of frequency within this in-class activity. This was not true for other in-class activities. Student presentations were a type of whole group discussion, in a sense, because after the students' initial explanation of their work the instructor *probed* students' thinking and asked questions to *Generate Discussion* by getting other students to contribute.

On Day 2 the students spent the last few minutes engaged in individual work on an optimization problem. The instructor only asked two questions during this time. One was of the *Requesting Validity* type and the other *Checking Understanding.* Both questions were answered immediately by a student and he asked both questions to one student.

Instructor T asked a variety of questions within four in-class activities, three of which showed up regularly in his observed classes. The in-class activities were similar in that they all contained *Checking Understanding* questions, and there was evidence of students answering more questions than the instructor during each. They were different in terms of the number and types of questions asked as well as the wait time used.

Summary

Instructor T asked more than 300 questions from 10 question types during his observed classes. *Gathering Information* questions had the highest frequency, yet the students were able to engage in deeper discussions about mathematics with his use of the *Generating Discussion, Probing,* and *Meaning/Relationships* types. It can be said that the instructor consciously gave students wait time although there was evidence of him not using wait time. The students answered the majority of the instructor's questions and usually did so within two seconds. At times it was because they had been called on by name to answer a question. The instructor answered a minimal amount of his own questions, but he never answered the questions he had given wait time.

Instructor T engaged with multiple students during class time using four in-class activities (small group work, whole group discussion, student presentations, and individual work). Similarities and differences existed between the in-class activities including the range of question types posed and the wait time provided. During whole group discussions he asked 9 question types and used wait time more often than not. A sharp contrast was observed as students worked in small groups on the warm-up as he only asked *Checking Understanding* questions and did not use wait time. Additionally, *Meaning/Relationships, Extending Thinking,* and *Inserting Terminology* questions were only used during whole group discussion and student presentations. The questions the instructor asked, the way he used wait time, and the in-class activities observed suggest that he aimed to engage them in discussions.

Case of Instructor C

Students in Instructor C's class learned Calculus I in an intimate setting. Seven tables, each made to seat two people, were organized in a horseshoe that was wider than it was long. Four tables were pushed together in the center of the horseshoe, and this is where the students sat as well as where the instructor placed his materials. Two dry erase boards were in the room; a large one on the front wall and one noticeably smaller on the wall adjacent left of it. Both were used by the instructor as well as the students. A computer sat on a table against the front wall to the right of the dry erase board. The computer and a projection system were used on one observed day for showing a video and projecting pages in the text.

This was a small class with only five students. The first day of observation there were four students in attendance, but on the other two days all of the students were present. The first two days began with the instructor writing example problems on the board and working through them for the students before giving them time to practice doing similar problems on their own. On Day 1 the instructor did the first four examples and gave students time to practice before showing them the final example. He then assigned additional practice. The second day was different in that Instructor C did all of the examples at once and then assigned problems to the students. The class began with a video on the final day of observation which was stopped once for a moment of lecture-discussion. Another short moment of lecture-discussion followed the video, and then the students were given practice time.

The practice problems were worked on individually and generally assigned a few at a time. Although the students worked alone on practice problems, they did sometimes consult with each other about their work. The instructor also occasionally talked to the students during this time. A relatively large amount of class time was used for student practice. Each student was responsible for putting at least one problem on the board daily and they were discussed. The students were hardly given the opportunity to explain their work because the instructor generally looked at the answers and talked through the problems himself. Instructor C asked the students questions along the way, and at times he simplified their work further. On Day 3, the students' practice problems included a few multiple choice. They expressed these answers verbally opposed to putting them on the board.

Instructor C did most of the talking during the observed class sessions, but he talked in longer spurts when working through the example problems. The sum of his first two talk turns on Day 1, for example, was over 1500 words separated by a student talk turn of just four words. The instructor spoke in smaller talk turns as he walked around the room and talked to students and as he went over the practice problems. There was evidence of larger talk turns during this time nevertheless (e.g., 347 words, 382 words, and 432 words). During these large talk turns the instructor explained, asked questions, answered them, and made comments. Instructor C covered one topic per day and engaged students in a few in-class activities. A general overview of each day can be found in Table 24.

Table 24: Overview of Instructor C's Class by Day

Day of Observation	Overview of Content	Classroom Context
Day 1	• Logarithmic differentiation	• Lecture-Discussion: Example Problems • Individual work on worksheet (#'s 1-4) • Whole group discussion • Lecture-Discussion: Example Problem • Individual work on worksheet (#'s 5-7) • Whole group discussion
Day 2	• Related Rates	• Lecture-Discussion: Example Problems • Practice Problems • Discussion of Problems
Day 3	• Minima and Maxima	• Video • Lecture-Discussion • Video • Lecture-Discussion

Table 24 Continued

		• Individual work on worksheet (#'s 1-5) • Whole group discussion • Individual work on worksheet (#'s 6-10) • Whole group discussion • Individual work on worksheet (#'s 11-15)

Questions by Type

Instructor C asked questions that fell into eight of 11 question types. This can be seen in Table 25 which shows the frequency of each question type during the observed class sessions. There was no evidence of the instructor using his questions to *Generate Discussion,* yet there was an ample amount of *Gathering Information* questions. In fact, more than 60% of the instructor's questions sought to *Gather Information* from the students. A little over 20% of his questions were devoted to the *Checking Understanding* and *Requesting Validity* types. There was also proof that Instructor C asked the students questions that moved beyond the surface with the *Probing* and *Meaning/Relationship* types. Other questions types occurred minimally, but they contribute to understanding the instructor's questioning.

Table 25: Question Types Used by Instructor C

Question Types	Frequency (Percentage)
1. Gathering Information	155 (63.0%)
2. Inserting Terminology	3 (1.2%)
3. Meaning/Relationships	14 (5.7%)
4. Probing	14 (5.7%)
5. Extending Thinking	3 (1.2%)
6. Context	2 (0.8%)
7. Checking Understanding	30 (12.2%)
8. Requesting Validity	25 (10.2%)
Total	246 (100%)

A look at the question types posed per day, displayed in Figure 14, shows that the *Gathering Information* type dominated Instructor C's questioning each of the observed days. There were at least five question types present on each day including *Gathering Information, Checking Understanding*, *Requesting Validity,* and *Probing.* Day 2 had the most question types present and also offered the most opportunities for students to engage more deeply with *Meaning/Relationships* questions. This may be partly attributed to the day's topic of discussion, related rates. The focus of this section will be on the question types that the instructor used most often. Attention will, however, be given to all other types in a single subsection as they contributed to the total picture of the instructor's questioning.

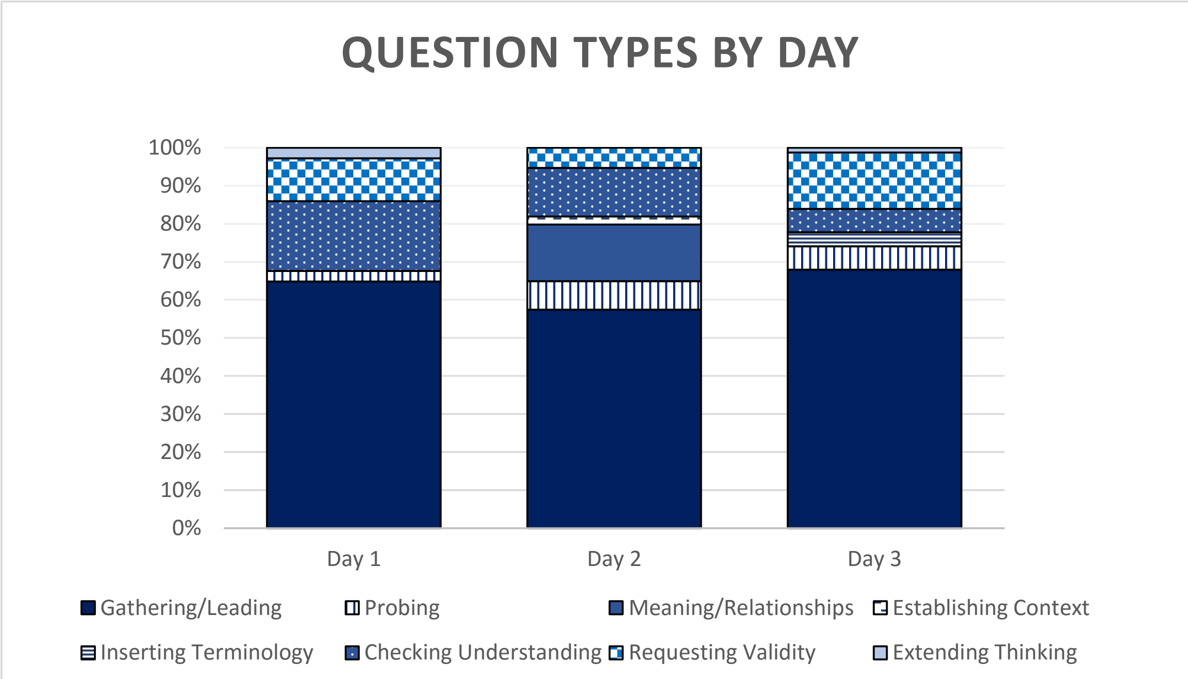

Figure 14: Instructor C's question types per day

Gathering information. A number of the questions identified as *Gathering Information* involved students finding something based on computation. In many cases it was derivatives that they were asked to find. These questions were present on the three observed days and required the students' use of known rules. The students were asked to find derivatives as a part of larger problems. On Day 1, for example, they worked on logarithmic differentiation which at times involves taking the natural logarithm of both sides of an equation and simplifying before differentiating both sides of it. This process included finding multiple derivatives, but none of them represented the final answer. He asked questions like, "What's the first derivative?", "So what's the derivative of secant?", and "What's the derivative of this?" He asked similar questions on Day 2 as they worked on related rates problems and on Day 3 when they were finding critical points. These questions were generally answered by the students, but they were occasionally answered by the instructor.

The students were asked questions that involved numeric computations on very few instances. Instructor C asked, "What is one half and one?" as he worked through simplifying the answer to a logarithmic differentiation problem. This was only observed at one other time. Consider the following excerpt which took place as the instructor went over a problem involving finding the minimum and maximum of a known function on a closed interval.

I: …Give me the value of this Buford. $f(2)$ what's the value?

S: $f(2)$ is 1.9.

I: 1.9 here. $f(1)$?

S: 2.1

I: 2.1. Which is 10 here?

S: 2.6

Here the instructor was asking the students to find the function values at the critical point and the endpoints of a given function which required performing numeric computations. All questions related to numeric computations were answered by students.

The focus of Instructor C's *Gathering Information* questions was sometimes to simply establish the answer(s) to problems. He was observed saying things like, "The answer is what?", "What's your answer?", and "What's your answer to number 6?" On Day 3, the students were given a worksheet on finding the minimum and maximum of functions. The first five problems were multiple choice, and Instructor C called students by name to provide an answer for each one. This discussion started with the following excerpt.

I: Ok. I see you working beyond 5. Ok so start with Brigitte. Brigitte what is your number one?

S: a.

I: Why a Brigitte? What made you decide that this is a, negative 3?

The students answered most questions that had a focus on answers, but as pointed out earlier the instructor sometimes answered his own questions. This excerpt also contained evidence of *Probing* which will be discussed in a later section.

Other questions the instructor asked to *Gather Information* required that students extract information from a problem. This was observed solely on Day 2. The instructor introduced related rates to his class by working through four examples. The final example said, "If x^2+y^2=25 and x'=-2, then what is y' if x=3 and y=-4?" Instructor C asked the students to extract information directly from that given in the problem with questions like "If they give us the x and the y, what is the constant?", "What is the value of my x?", and "What's the value of my change of the x?" It was observed that the students normally answered these questions.

Instructor C also asked his students if they remembered something each day as a means of *Gathering Information.* For example, as the instructor worked through a logarithmic differentiation problem he showed the students an approach to simply an expression using what some people call "cross multiply." He asked the students, "Remember that?" after he finished demonstrating the approach on the board. In another example he asked the students, "Remember implicit?" He wanted to know if the students remembered implicit differentiation because it is a vital part of related rates which was the topic of Day 2's lesson. These questions went unanswered, but the students may have answered them nonverbally. It could have been that they did remember and did not feel the need to respond verbally.

At times Instructor C's *Gathering Information* questions simply required a quick response. For example, "Can we simplify more or no more?", "Did you substitute?", and "Is the lowest point a solid or closed dot?" There was also a couple of questions that asked students to identify something based on known information. One example took place during a related rates problem when the instructor asked, "What's the equation of that triangle?" *Gathering Information* questions, as a whole, were used to collect different kinds of information from the students.

Requesting validity. During each of the observed classes, Instructor C asked students to validate claims he made while explaining problems. At times the questions were straight forward and asked the students if something was "Correct." The following excerpt from Day 3 is one of less than 10 examples using "Correct," but it's one of two in terms of receiving a student answer.

I: Ok what's the first derivative of tangent x?

S: Uh secant squared x.

I: Secant squared x and equal it to zero. Secant is one over cosine squared x. Is that correct?

S: Yes.

There was also evidence of a statement followed by "Correct?" that functioned the same way. The instructor said, "So actually this is about two, negative two x minus two minus x. Correct?" in regard to simplifying a problem involving logarithmic differentiation. This question did not receive a student response.

Instructor C used statements followed by “Right?” to *Request Validity* of his claims as well. The students answered many of these questions with a simple phrase of agreement. Consider this short passage.

I: …What’s your answer to number 6?

S: 33.9.

I: 33.9. Yea. Which is the volume. Right?

S: Yes.

The instructor asked a student for her answer to a particular related rates problem. After she shared her answer, the instructor repeated and confirmed it. He then made the claim that the answer was the volume and asked for validation. There was minimal evidence of students expressing disagreement.

Other questions of the *Requesting Validity* type did not use the words “correct” or “right,” yet they still asked students to confirm something. For example, “And if we simplify them and put them together that’s the same thing as what negative three x?” He was asking them to validate that simplifying produced $-3x$. This particular question did not yield a verbal response, yet *Requested Validity* questions as a whole engaged students in expressing their agreement.

Checking understanding. Instructor C did a lot of explaining at the board as he worked on sample problems and went over the students’ work from their practice problems. He made it a point to ask the students if they had questions about the problems on the board before moving on to discuss others. In fact, he did this at some point in regard to every problem they went over. These questions involved students in examining their understanding of what had been explained as well as what had been written on the board. Some examples of the questions Instructor C asked of the *Checking Understanding* type are “Ok, so any questions?”, “Questions before I will proceed?” and “Do you have a question about numbers 5, 6, and number 7?” The bulk of questions identified as this type used the word “question,” but there were instances that did not.

A couple of times the instructor explicitly asked the students if they understood. For example, “Did you understand number one?” In one situation he asked the students if they had a “comment” on a problem. There was also a few cases of a statement followed by “Alright?” Consider the excerpt below which took place as the students reported on their individual part to solve a related rates problem.

> I: Ok Tresca this is your good start. This is your x prime, this is your y prime. We need r prime. Of course the r prime is the first derivative. So what’s the first derivative? If you follow that it is almost the same thing. Alright?
>
> S: Yea.

Here a student was asked to find the derivative. Before given a moment to do what was asked, the instructor pointed out that it was almost like an example they had gone over and said “Alright?” The student’s response, although simple, said that she understood what the instructor was saying. This is one of five occasions that a student responded to a *Checking Understanding* question. The other replies were “no,” but there was one student question.

The questions in the category were generally followed by an instructor comment or explanation. It is likely that this was the case because these questions did not require a verbal response. The instructor may have been paying attention to nonverbal answers. It can be gathered that he expected students to respond in some capacity to these questions because there was evidence of him asking the same or similar questions back to back. It is assumed that when the students did not give him any indication that they understood what was going on he asked them again. *Checking Understanding* questions required that students think about their comprehension of what had been explained or written, but they did not dig into students' thinking like the *Probing* type.

Probing. The questions Instructor C asked rarely followed-up on students' thinking, but there was evidence of *Probing* questions on each of the observed days. Many of the instructor's *Probing* questions involved the use of "why?" A couple illustrations transpired as the students provided their answers to multiple choice questions about minimum and maximum and asked the students why they made a decision. The short excerpt below shows one of those examples.

I: …How about number 3 Ernie. What is your choice for number 3?

S: a.

I: You used a. Why a?

S: Cause x equals negative one when the graph is at four is the highest point.

Another *Probing* question asked why something was done as they discussed a related rates problem. After they found the derivative of the appropriate function, the student asked a question which started the dialogue below.

S: So would that be a?

I: What a?

S: Where did you get x squared from?

I: That's the given. That's the base times the height.

S: Ok, but I thought the equation was one half base times height.

I: Yes, but why is it that we marked it x and x?

S: Cause they are moving at the same rate.

This passage contains two questions that *Probed* the same student's thinking. The first question "What a?" was asking the student to elaborate. She was confused about where x squared came from in the equation, so the instructor later dug into her thinking by asking "Yes, but why is it that we marked it x and x?"

In addition to *Probing* questions that asked students to explain why or elaborate, the instructor asked a couple of questions that asked students how they did something. They did not occur often, but the students answered these questions more often than they did not. *Probing* questions give students opportunities to explain their thoughts and think about mathematics more deeply.

Meaning/Relationships. Questions of the *Meaning/Relationships* type were observed minimally and were only seen on Day 2 as they talked about related rates. Instructor C began talking about one problem by saying, "So read the problem number five and what is your possible figure that we can draw here?" A student answered, "A right triangle" indicating that they could create a graphical representation of what was given in text. The instructor also asked the students to identify the given information which required identifying what was a rate and what was a constant. Pinpointing where the given belonged in the graphical representation also allowed students to make meaning. They were asked, "So where is the five miles in the right angle?" Instructor C also asked, "So it means the prime the rate of change of the, what distance? Find the rate of the distance of what?" to see if they had made meaning of what the problem was asking them for. The students did not always answer these questions, but they stand as moments they had to make meaning in Calculus I.

Additional question types. Instructor C hardly asked questions of the *Inserting Terminology*, *Establishing Context*, and *Extending Thinking* types. In fact, these question types had individual frequencies of three or less. An example of each can be found in Table 26. It is important to highlight these questions because they too were a part of the questions the instructor asked.

Table 26: Examples from Instructor C's Question Types with Low Frequency

Question Type	Frequency	Example
Inserting Terminology	3	What do we have here if we have the critical value or critical point; what's this value?
Establishing Context	2	Remember in the gulf?
Extending Thinking	3	Now what happens if we will use logarithmic differentiation and there is no ln?

The *Inserting Terminology* example was asked as the instructor pointed at the vertex on a graph of a function that was "facing up" and continued without bound. They had been talking about absolute as well as local minima, so he asked this question to assure that students used the appropriate terminology. The *Establishing Context* question was asked while the instructor went over an example related rates problem. The problem was very procedural as it had no reference to a real-world situation. The instructor told the students the problem could be an “oil spill” and asked if they remembered in the gulf because of the relatively recent BP oil spill. Instructor C *extended* the students’ thinking when he asked them to think about how they would perform logarithmic differentiation when there was not a natural logarithm in the given function.

Summary. Of the eight question types observed in Instructor C’s questioning *Gathering Information* was used most frequently to engage students in discourse. The instructor did the bulk of the explaining during the observed class sessions and used his questions to *Check Understanding* and *Request Validation* accordingly. Questions were not often used to probe into students’ thinking or to make meaning of ideas. Regardless of the type of questions asked, the students often answered them.

Wait Time

Instructor C’s questions were followed by each category of wait time during the three observed class sessions. Figure 15 shows his use of wait time by percent. Interestingly, the instructor provided some form of wait time after his questions more often than he did not. *Minimal Wait-Time, Wait-Time,* and *Extended Wait-Time* collectively represented 35% of the instructor’s questions compared to 28% identified as *No Wait-Time.* It can also be seen that the largest percentage of the instructor’s questions were promptly responded to by the students.

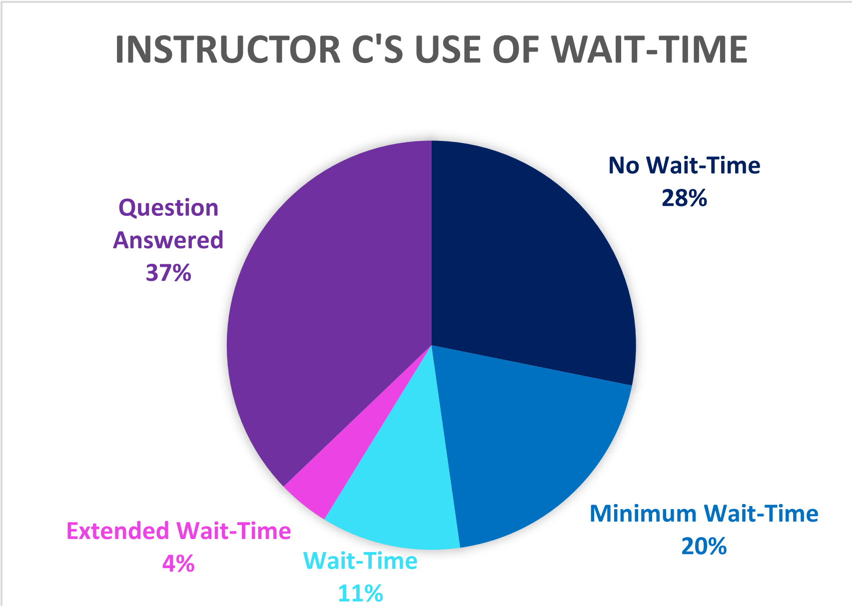

Figure 15: Instructor C's use of wait time

It can be observed in Figure 16 that there was evidence of each wait time category on all observed days. The Instructor provided no wait time most often on Day 1, however it was also the day where questions were more frequently given some form of wait time. It was more common for the students to quickly respond to the instructor's questions on Days 2 and 3. There was an increase in the percent of questions the students immediately responded to over the course of the three observed days, and there was a decrease in the percent of questions followed by *Minimal Wait-Time.*

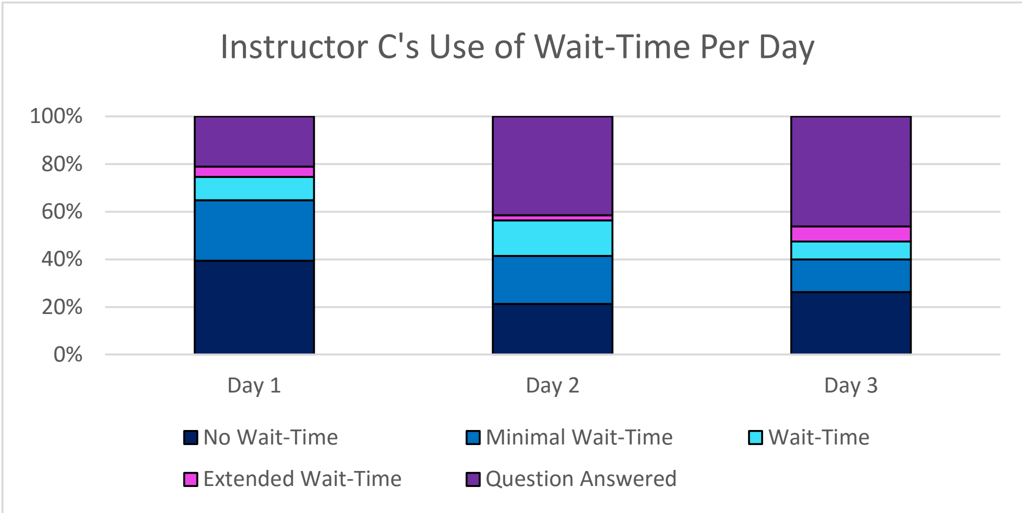

Figure 16: Instructor C's use of wait time per day

The frequency that each question type was coded as *No Wait-Time, Minimal Wait-Time, Wait-Time, Extended Wait-Time,* and *Immediate Student Response* can be found in Table 27. It shows that Instructor C did not use a particular kind of wait time for specific question types. The majority of the question types had multiple wait time categories present. *Gathering Information* and *Checking Understanding* were the only question types with evidence of each wait time category. An attention grabbing finding can be extracted from this table; there were five question types where wait time was provided as often or more often than not. Specifically, wait time and no wait had equal presence for *Checking Understanding* and *Probing* questions; *Gathering Information*, *Meaning/Relationships*, and *Orienting/ Focusing* questions each had more wait time than no wait time. It is worth pointing out that *Minimal Wait-Time* was the most common making up close to 58% of the questions given some form of wait time.

Table 27: Instructor C's Use of Wait Time by Question Type

	No Wait	Minimal Wait	Wait	Extended Wait	Immediate Student Response	Total
Gathering/ Leading	40	32	18	6	59	155
Requesting Validity	7	4	0	1	13	25
Checking Understanding	13	5	5	3	4	30
Meaning/ Relationships	2	4	3	0	5	14
Inserting Terminology	0	0	0	0	3	3
Probing	4	3	1	0	6	14
Extending Thinking	2	0	0	0	1	3
Establishing Context	1	0	0	0	1	2
Total	69	48	27	10	92	246

Questioning without wait time. All of the question types in Instructor C's observed class sessions were at some point given no wait time with the exception of three, namely, *Linking/Applying, Orienting/Focusing,* and *Inserting Terminology.* A little over half of these questions were followed by an instructor explanation or comment. Another 30% of the questions were answered immediately by the instructor. An instructor question came after the remainder of the questions observed with no wait time.

Questions that were followed by an instructor explanation or comment were of the *Gathering Information, Checking Understanding, Requesting Validity, Extending Thinking, Probing,* and *Meaning/Relationships* types. The *Gathering Information* and *Checking*

Understanding types had the largest presence. The *Gathering Information* questions generally asked students if they remembered something. For example, Instructor C asked "Remember that in the law of exponents?" after pointing out that the exponent would change signs if he moved the denominator to the "top" as he worked through a problem using logarithmic differentiation. He then reiterated the fact that, "the denominator can be the numerator…" All of the *Gathering Information* questions followed by an instructor explanation or comment went unanswered. Let it be known that a simple yes or no would have sufficed as an answer to most of these questions, as such the instructor may have given attention to nonverbal responses.

Checking Understanding and *Requesting Validity* questions were similar to the *Gathering Information* questions described in that they did not receive nor require a verbal response. After the instructor explained a generic related rates example, he asked "Questions before I will proceed?" The students had less than two seconds to check their understanding of the example before the instructor went on to explain that there are numerous related rates problems, and the only way they can "understand how to attack any problem is to understand this; to make sure which one is constant and which one is not…" Similar questions were asked after other problems were explained. Instructor C requested that the students validate claims on a few occasions with questions like "Is that correct?" and "Can you check?" Again, the students did not verbally respond to these questions. Questions identified as *Probing, Meaning/Relationships,* and *Extending Thinking* also went unanswered when they were followed by an instructor explanation or comment. This resulted in missed opportunities for students to think deeply.

Instructor C promptly answered 30% of the questions identified as no wait time. These questions were primarily of the *Gathering Information* type and represented questions the students should have been comfortable answering at that point in a Calculus I class. He asked and answered questions like, “So what would happen if I multiply that with the ln?” and “So the derivative, the derivative of x squared plus y squared is equal to 25 is what?” which involved students rehearsal of known facts and procedures. There were also single occurrences of the instructor immediately answering an *Establishing Context, Probing,* and *Meaning/Relationship* question.

Questions coded as *No Wait-Time* that did not receive an instructor comment, explanation or answer, were followed by an instructor question. This was the case for less than 20% of the questions Instructor C was the first to respond to. In a couple of instances he asked a question, paused less than two seconds, and then repeated or asked a similar question. For example, on Day 2, he said “Did you understand number one? [Less than 2 seconds] No questions? [2-3 seconds] So number two. Maggie or James.” Here he asked two *Checking Understanding* questions. Neither got a verbal student response, but there is reason to believe he expected a response. Other examples were identified in the *Gathering Information, Probing,* and *Requesting Validity* types. All questions with no wait time followed by another question went unanswered, but the subsequent question was sometimes answered.

Instructor C responded to his questions a number of times in less than two seconds. Questions with no wait time were observed in the majority of the question types and were often followed by an instructor explanation or comment. The instructor answered a noticeable number of these questions, but also asked back to back questions on occasion. The questions that were not answered by the instructor went unanswered by the students. Nonverbal answers would have sufficed in some cases and may have been acknowledged by the instructor.

Questioning with wait time. Similar to no wait time, most of the question types were followed by some form of wait time. There was no evidence of *Minimal Wait-Time, Wait-Time,* or *Extended Wait-Time* for the following types: *Linking/Applying, Inserting Terminology, Extending Thinking,* and *Establishing Context.* This can be attributed to low question frequencies and the fact that the students generally responded to these questions within two seconds. A student answer was the most common response to Instructor C's questions followed by wait time. The frequency of responses that were an instructor question was only one less than a student answer. In spite of this, the instructor was the supreme responder when his comments, explanations, and answers are considered.

The students answered *Gathering Information, Probing, Checking Understanding, Requesting Validity,* and *Meaning/Relationships* questions when wait time was provided. This was normally done within two to three seconds and with questions of the *Gathering Information* type. There was evidence of questions with *Wait-Time* (3-5 seconds) and *Extended Wait-Time* (more than 5 seconds) that yielded a student answer. Instructor C asked "What is the derivative of a constant?" and "…can you explain that?", for instance, and received a student answer after waiting between two and three seconds. He asked "What's the equation we can create from there; from the graph?" and waited three to five seconds before getting a student answer. "Number two. What's the maximum?" and "What's the value of x to find the minimum?" were the only two questions that students answered after the instructor waited for more than five seconds for a response.

Similar to questions that the students answered, Instructor C responded with another question most often after waiting two to three seconds. There were instances followed by more time than that. The initial question usually went answered, but there was evidence of one question being answered by a student. The instructor asked, "So what's the derivative of secant?" and waited two to three seconds before leading the students with "$Secx$?" A student immediately answered $tanx,$ finishing off the derivative of $sec(x)$ which is $secxtanx.$ In a couple of instances Instructor C asked the same question after waiting and getting no response. For example he asked, "Do you have a comment on that number seven?" in reference to a logarithmic differentiation problem a student had worked on the board. The instructor waited more than five seconds before asking the question again. He waited an additional three to five seconds before making his own comments in reference to the problem.

The instructor followed his questions with comments and explanations after a variety of question types. They were mainly *Checking Understanding* and *Gathering Information* questions with a similar presence of *Minimal Wait-Time* and *Wait-Time.* The instructor asked "Do you have any questions?", "… Remember this?", and similar questions that did not necessitate a verbal response and paused for more than three seconds before adding a comment or explanation. There were also times where he asked a question, waited, commented/explained, and asked the same question. Consider the following instructor quote, "Where is the other four roots? [2-3 seconds] Remember there are five; there are five roots, but the graph shows there's only one touching the x axis which is one of the roots. Where is the other four?" A student immediately answered the question when he asked it the second time, in this particular case.

Instructor C answered a little over 10% of his questions followed by some type of wait time. He answered questions including but not limited to the following: "So what's the rule of multiplication?", "What's the original equation to that number 10?", and "What are you going to do with the one over y?" These questions were primarily of the *Gathering Information* type. It is likely that the instructor answered his questions after providing wait time when he felt the students either were not going to answer or were taking too long to answer.

There was a range of wait time provided in Instructor C's observed Calculus I sessions, but wait times of two to three seconds were common. The instructor was the first to respond to most of these questions, however, the students actually answered most of the questions followed by wait time. Questions were the most common instructor response when wait time was provided, yet the instructor responded with explanations and comments as well as an occasional answer.

Summary. Wait time in Instructor C's class took precedent over questions without wait time suggesting that students were given time to respond or at least think more often than not. The students answered numerous questions followed by wait time, yet it was common for them to respond to the instructor's questions within 2 seconds. Instructor C answered his own questions more often when no wait time was provided compared to when it was. On the other hand, he asked back to back questions less often when there was no wait time versus when there was. As previously noted *No Wait-Time*, *Minimal Wait-Time*, *Wait-Time* and *Extended Wait-Time* was observed after a variety of question types, however, there was not a specific wait time utilized with a particular question type.

Classroom Setting

Instructor C's observed classes entailed three in-class activities, namely, lecture-discussion, individual work, and whole group discussion. Lecture-discussion was observed during the first hour of class as the instructor worked through example problems. It only exceeded this time on Day 1 when he had students practice problems before moving forward with his final example. It is important to note that the instructor never spent an entire hour transmitting information. For example, on Day 3 the first hour included a video and lecture-discussion. The students also began their practice problems during the first hour each day.

Thirty-five percent of the instructor's questions were asked during lecture-discussion. Further, seven of the eight observed questions types were utilized when Instructor C used lecture-discussion. There was no evidence of *Meaning/Relationships* in the course of lecture-discussion, but there was a bulk of *Gathering Information.* The instructor also asked questions of the *Checking Understanding*, *Requesting Validity*, *Probing*, *Extending Thinking*, *Inserting Terminology*, and *Establishing Context* types. Questions during lecture-discussion were followed by no wait time more often than wait time, and the instructor was generally the first to respond. The instructor's responses did not frequently answer the questions. Instead, it was generally a comment/explanation or subsequent question that followed Instructor C's questions as he utilized lecture-discussion.

The students spent time each day working on practice problems individually. The instructor asked the least number of questions while students were engaged in this in-class activity. Like lecture-discussion there were multiple question types present, but there was one less question type present during individual work. Instructor C did not ask any *Extending Thinking* or *Establishing Context* questions while the students worked on their practice problems. There was, however, evidence of *Meaning/Relationships* questions that were not seen in lecture-discussion. This resulted in students being asked a larger number of questions that pushed them to think when working individually. Unlike lecture-discussion, students were generally the first to respond to the questions posed when they worked individually. Wait time was also provided more often than not during individual work. This is the opposite of what occurred during lecture-discussion, but it is consistent with the questions posed during whole group discussion.

The practice problems that students worked on individually were at some point the topic of whole group discussion in Instructor C's three observed classes. This particular in-class activity housed almost half of the questions posed. The instructor posed the same types of questions during whole group discussion as he did when the students worked individually. Again, students were given an opportunity to think about mathematics more deeply because of the presence of *Meaning/Relationship* questions in addition to *Probing.* It is worth noting that the students were as likely to respond to the instructor's questions as he was, however, the students responses were answers to the questions more often than the instructor's.

Instructor C engaged students in the same in-class activities daily over the course of three observed days. On each day, the smallest number of questions were posed during students' individual work. Whole group discussion all in all included the most questions, but on Day 1 the instructor asked more questions during his lecture-discussion than any other in-class activity. It was observed that *Gathering Information, Checking Understanding, Requesting Validity, Probing,* and *Inserting Terminology* questions were a part of each in-class activity. *Meaning/Relationships* questions were limited to individual work and whole group discussion whereas *Extending Thinking* and *Establishing Context* questions were only asked during lecture-discussion. As previously mentioned, *Meaning/Relationships* questions only occurred on Day 2 when related rates was the topic of discussion suggesting that the content may have had something to do with the instructor asking this type of question opposed to the in-class activity.

The answers to the questions posed were most often provided by the students despite the in-class activity. The instructor was, however, the first to respond to most of his questions during lecture-discussion. Wait time was provided during each in-class activity with *Minimal Wait-Time* being most common. Additionally, the instructor paused for wait time more often than not during both individual work and whole group discussion.

Summary

Overall, the questions asked in Instructor C's observed Calculus I sessions *Gathered Information* from the students. This was true for all three observed days as well as for each in-class activity. His questioning as a whole limited the students' opportunities to engage in discourse that went beyond short responses and/or correct or incorrect answers because there was minimal evidence of questions that functioned to probe students thinking and explore meaning/relationships. Instructor C's students answered many questions immediately, yet it can be concluded that this was not because they were restricted to answering within a two second time frame. Wait time was given slightly more than no wait time, however, this was generally a pause of two to three seconds. The instructor surpassed what seemed to be his norm when he asked 14 *Meaning/Relationships* and seven *Probing* questions on one day and when he exceeded wait times of five seconds which only happened 10 times.

Cross-Case Analysis

The remainder of this chapter is devoted to describing the similarities and differences in the case instructors' use of mathematical questions. As previously noted, five Calculus I classes were observed for approximately 6 consecutive hours of instruction. In alignment with the individual cases that have been described, the focus here is on the types of questions the instructors asked, the wait time they used, and the classroom setting(s) that the questions were asked in.

Questions Type

Instructors D and F used all of the question types, as defined in the adapted categories of questions (Boaler & Brodie, 2004; Stolk, 2013) as seen in Table 28. Seven question types were observed across all five instructors, namely, *Gathering Information, Requesting Validity, Checking Understanding, Probing, Meaning/Relationships, Inserting Terminology,* and *Extending Thinking.*

Table 28: Question Types Used by Each Instructor

	Instructor				
Question Type	D	P	T	F	C
Gathering Information	139 (49.1%)	235 (46.3%)	106 (31.7%)	524 (72.2%)	155 (63.0%)
Requesting validity	40 (14.1%)	86 (16.9%)	74 (22.2%)	32 (4.4%)	25 (10.2%)
Checking Understanding	54 (19.1%)	48 (9.4%)	60 (18.0%)	41 (5.6%)	30 (12.2%)
Probing	7 (2.5%)	33 (6.5%)	37 (11.1%)	48 (6.6%)	14 (5.7%)
Meaning/Relationships	2 (0.7%)	30 (5.9%)	15 (4.5%)	15 (2.1%)	14 (5.7%)
Inserting Terminology	2 (0.7%)	9 (1.8%)	5 (1.5%)	5 (0.7%)	3 (1.2%)
Establishing Context	6 (2.1%)	5 (1.0%)	0 (0.0%)	2 (0.3%)	2 (0.8%)
Generating Discussion	28 (9.9%)	59 (11.6%)	33 (9.9%)	34 (4.7%)	0 (0.0%)
Linking/Applying	1 (0.4%)	0 (0.0%)	0 (0.0%)	8 (1.1%)	0 (0.0%)
Extending Thinking	3 (1.1%)	2 (0.4%)	2 (0.6%)	3 (0.4%)	3 (1.2%)
Orienting/Focusing	1 (0.4%)	1 (0.2%)	2 (0.6%)	14 (1.9%)	0 (0.0%)
Total	283 (100.1%)	508 (100.0%)	334 (100.1%)	726 (100.0%)	246 (100.0%)

Linking/Applying, *Extending Thinking*, *Orienting/Focusing*, *Inserting Terminology*, and *Establishing Context* question types were asked in low numbers for each of the instructors that utilized them. As a result, the data was reorganized into fewer question types and used to perform statistical analysis. The *Establishing Context* question type was placed under the *Gathering Information* question type as supported by Stolk (2013). The remaining question types of low frequency were grouped together and recognized as the *Collapsed Category* based on the fact that the instructors' collective use of the individual question types did not exceed 6% (e.g., *Orienting/Focusing* made up 3.1% of the instructors' overall questioning). The frequencies of the seven reorganized question types can be found in Table 29.

Table 29: Reorganized Question Types Used by Each Instructor

	Instructor				
Question Type	D	P	T	F	C
Gathering Information	145	240	106	526	157
Meaning/Relationships	2	30	15	15	14
Probing	7	33	37	48	14
Generating Discussion	28	59	33	34	0
Checking/Understanding	54	48	60	41	30
Requesting Validity	40	86	74	32	25
Collapsed Category	7	12	9	30	6
Total	283	508	334	726	246

The relationship between the number of questions asked and the seven question types was investigated. First, one-way Analysis of Variance (ANOVA) was used to test the following hypotheses:

H_0: The mean number of questions asked is the same for each question type

H_1: The mean number of questions asked is different for at least one question type

Based on the ANOVA results (F=7.02, p=.0001), the average number of questions differs for at least one of the question types. Further analysis (via a Tukey's multiple comparisons test) revealed that on average, instructors asked *Gathering Information* questions more than any other type of question, and instructors asked the other six types of questions (*Meaning/Relationships, Probing, Generating Discussion, Checking/Understanding, Requesting Validity, Collapsed Category*) with the same frequency.

It should be noted that there were some violations of the ANOVA assumptions; therefore non-parametric methods were also utilized to test for differences among number of questions based on question type. These methods are based on the ranks of the data; thus they are not as statistically powerful in their detection of differences. However, they do not rely on the assumption of normally distributed data. More information on the advantages and disadvantages of non-parametric statistical methods can be found in Hollander, Wolfe and Chicken (2014) and Dallal (2012). Boxplots of Wilcoxon Rank Sum Scores based on question type where 1=Gathering Information, 2=Meaning/Relationships, 3=Probing, 4=Generating Discussion, 5= Collapsed Category, 6= Checking Understanding, and 7=Requesting Validity can be found in Figure 17.

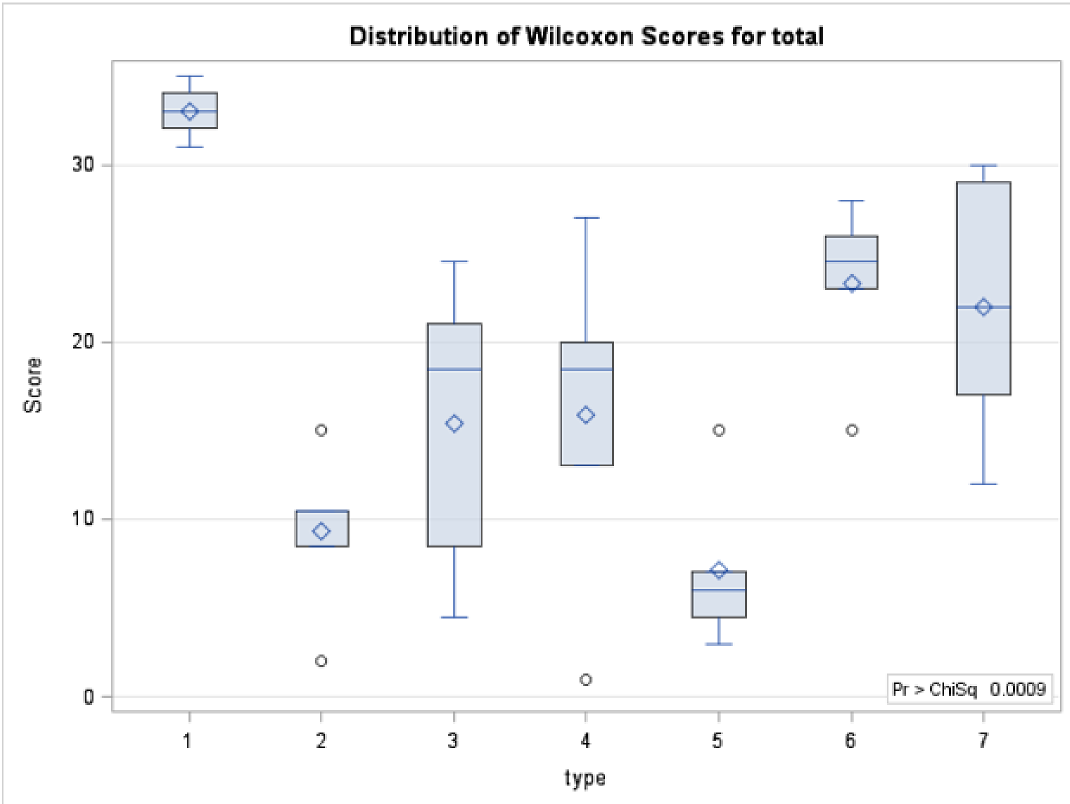

Figure 17: Boxplots of Wilcoxon Rank Sum Scores based on question type

A Kruskal-Wallis test showed that there was a statistically significant difference among the number of questions asked based on question type, $\chi^2(2) = 22.64$, p =.0009. A Kruskal-Wallis-based multiple comparisons procedure (Elliott and Hynan, 2011) was performed to identify where the difference(s) were. It revealed a significant different between *Gathering Information* and *Meaning/Relationships* (Rank Sums=165 and 46.5, respectively) as well as *Gathering Information* and the *Collapsed Category* (Rank Sums=165 and 35.5, respectively). Thus, this test, along with the boxplots in Figure 17 revealed that instructors asked more *Gathering Information* questions than both *Meaning/ Relationships* questions and the *Collapsed Category* questions. Unlike ANOVA, the Kruskal-Wallis test did not detect that instructors asked more *Gathering Information* questions than *Probing, Generating Discussion, Checking Understanding* and *Requesting Validity* questions.

The data was further explored using a chi-squared test to determine whether question type depends on instructor (H_0=question type and instructor are independent vs. H_a=question type and instructor are not independent). *Generating Discussion* was omitted from the analysis because Instructor C did not use that question type creating a violation for the chi-squared test. The results indicate that question type depends on instructor, $\chi^2 = 248.613$, p=<.0001. Practically, this indicates that different instructors engage their students with different types of questions.

Question types common to each instructor. *Gathering Information* questions were asked with the highest frequency by each of the instructors during this study. The questions did not always ask students for the same kind of information, yet most of them required that students recall facts and procedures. Common to each of the instructors *Gathering Information* questions were those that asked students to perform some kind of computation. Many of these questions asked for derivative computations. There was, however, minimal evidence of questions about arithmetic computation from the instructors. Questions about computation were generally a part of a larger problem as seen with finding derivatives while working on L'Hôpital's rule, optimization, and related rates. The instructors also asked students to state known facts like formulas, theorems, and rules as they worked on problems around various topics (e.g., area of a circle, Pythagorean Theorem, product rule).

All of the instructors except Instructor C asked students to state procedures or next steps in them. For instance, Instructors T, F, and P each asked their students for the "steps" in the optimization procedure. Similarly, Instructor D asked her students to state the procedure they previously used for finding the minimum of a function. Instructor's C and P were also observed asking students to extract information directly from problems as a means of *Gathering Information.* None of the other instructors used this question type in that exact way, yet Instructors T and F asked questions that may be viewed similarly. For example, the students in Instructor T's class were asked to identify coefficients and the degree of numerators and denominators of written functions, and Instructor F asked her students to identify where the graph of the sine function was increasing, decreasing, and constant. These questions were similar to the questions posed by Instructors C and P in that they asked students to extract information, but different because they required students' recollection of what terminology means in order to answer the question(s).

Aside from asking students to recall and state known facts and procedures, a few instructors asked their students 'if' they remembered something. Instructors D, F, and C were observed *Gathering Information* in this way. Instructor C did this regularly. These were questions that could be answered with "yes" or "no" like, "…remember the commutative law?", which was asked by Instructor D. Instructor F *Gathered Information* about "what" her students remembered. This was observed when she explicitly asked her students what they talked about in the previous class session. Instructor P also *Gathered Information* with his questions in a distinct way. He asked his students in the moment questions about what to do next or how to do things that could be answered based on prior knowledge. The answers to problems were at times the focus of *Gathering Information* questions asked by Instructors T, P, D, and C. These questions were generally asked during the class session that the problem was introduced, yet Instructor P was observed asking his students if they had an answer to a problem they started the previous day.

Requesting Validity and *Checking Understanding* were also asked numerous times each day by the instructors during their observed class sessions. Unlike *Gathering Information* questions, these types of questions were pretty much used by the instructors in one way. The two question types functioned differently, yet they generally only necessitated a yes or no answer. *Requesting Validity* questions asked the students to determine if they agreed or disagreed with the claims that were made. The students were asked to validate the instructors' claims in all of the observed classes, but Instructors T, P, and D also asked their students to validate the claims of their classmates. The claims that the students were being asked to validate were generally verbal, but there was evidence of Instructors P and F *Requesting Validity* of ideas that had been written on the board. *Checking Understanding* questions were not about the students' agreement; instead they challenged the students to examine their understanding of explanations and problems. These questions were asked in reference to the explanations offered by the instructors, however, Instructors P and T also asked these questions about student explanations. Both question types gave students an opportunity to make their opinions explicit.

The instructors in this study gave their students opportunities to engage at deeper levels with their use of *Probing* and *Meaning/Relationship* questions, however, some instructors used them in higher numbers. For example, approximately 11% of Instructor T's questions were identified as *Probing* whereas around 3% of Instructor D's questions were of this type. *Probing* questions asked students to explain their thoughts and often involved “how” or “why.” Each of the instructors *Probed* into students’ thinking by asking how they found an answer. They also *Probed* for students thoughts on why or how they knew something. Further, there were instances of the instructors *Probing* their students for additional information. These questions did not ask how or why, but were more about getting the students to say more. Uniquely, Instructors F, T, and P *Probed* their students with questions that asked for the meaning of something. Questions of this type were asked daily by all instructors except Instructor D who asked *Probing* questions four of seven days.

Meaning/Relationship questions were also asked by all of the instructors. Questions of this type were consistently observed in low numbers, and Instructors D and C only asked them on one day. The students were asked to make links between representations (e.g., determining the where something shows up in a graphical representation or diagram) when their instructors asked these types of questions. Instructor D only asked two *Meaning/Relationships* questions and both were of this nature. The other instructors’ questions of this type included asking students to make meaning of ideas in the context of problems. For example, this was seen when Instructor F asked “So what is dr/dt?” to get students to make meaning of the symbolic representation of a derivative in a specific related rates problem. Additional questions asked the students to make links between mathematical ideas as seen in Instructor T’s class when the students established links between a function and its first and second derivative. The main purpose of these questions was for students to make their own meaning and establish relationships.

Two additional question types were asked by each of the instructors; *Inserting Terminology* and *Extending Thinking.* These questions were asked in very low frequencies.

In most cases the frequencies were below five. Instructor P, was an exception with nine *Inserting Terminology* questions which he asked across five observed days. *Inserting Terminology* questions checked to see if students knew the appropriate terminology, but they also gave students a chance to correct themselves when terminology was used incorrectly. The instructors asked students to think about how ideas would apply to situations beyond those which they had discussed with *Extending Thinking* questions. For example, asking what happens when numbers are changed in a problem or if a different question is asked in regard to the same information.

Question types not common to all instructors. There were four question types that were not utilized by all of the instructors, namely, *Linking/Applying*, *Establishing Context*, *Orienting/Focusing* and *Generating Discussion.* Instructors D and F were the only instructors to ask *Linking/Applying* questions, and they both did so sparingly. Instructor D linked mathematics to other fields of study with her *Linking/Applying* questions, whereas Instructor F asked her students to come up with examples by applying their knowledge. Each of the instructors except Instructor T, used their questions to *Establish Context.* Again, these questions were observed in low frequencies. They were used the same by the instructors who asked them. *Orienting/Focusing* and *Generating Discussion* questions were posed by each of the instructors minus Instructor C. The former type of questions also occurred in low numbers, and they were always utilized to focus students' attention on key aspects during problem solving. *Generating Discussion* questions were asked with higher frequencies and there were similarities and differences in how they were used.

Each of the instructors that posed questions to *Generate Discussion* did so by asking students to share their thoughts and ideas about topics or problems that had yet to be discussed in class. Instructors F and T also used this question type to solicit contributions from other students. This was only observed a few times in Instructor F's class, but Instructor T did so consistently and he often called students by name when he *Generated Discussion* in this way. Instructors D and P *Generated Discussion* differently by asking their students what they wanted to do during class. All of these questions elicited students' thoughts, yet they did not always produce discussions beyond the answer to the question itself.

Summary. A look across instructors shows that there was a range of question types posed during the observed Calculus I classes. *Gathering Information* questions were asked with the highest frequency, yet there was evidence of questions that engaged students beyond the known including *Probing, Meaning/Relationships,* and *Generating Discussion.* Many questions types were observed in low frequencies and were utilized by the instructors in the same way (e.g., *Inserting Terminology, Establishing Context,* and *Extending Thinking)*. On the other hand, three question types displayed noticeable variation, namely, *Gathering Information, Probing,* and *Generating Discussion.* It was established after reducing the number of question types that there was a significant difference found among the number of questions posed by type. Specifically, ANOVA results showed that *Gathering Information* questions were asked significantly more than any other type of question. The instructors gave students different opportunities to engage in discourse with the questions they posed as it was determined with the chi-squared test that question type (*Generating Discussion* omitted) depends on instructor.

Wait Time

The instructors of the observed Calculus I classes utilized a range of wait time during this study. All of the coded wait time categories, namely, *Immediate Student Response* (student response <2 seconds), *No Wait-Time* (<2 seconds), *Minimal Wait-Time* (2-3 seconds), *Wait-Time* (3-5 seconds), and *Extended Wait-Time* (>5 seconds) were utilized at some point during six consecutive hours of instruction regardless of who was being observed, however, there were similarities and differences in the instructors' use of wait time. For the most part, each wait time category was evident daily per instructor. Instructor D was the exception with no instances of *Extended Wait-Time* on Day 2 of 7.

Immediate Student Response occurred with the highest frequency compared to other wait time categories for all of the instructors except Instructor D as seen in Table 30. Instructor D used *No Wait-Time* with the highest frequency behind her questions. This means that Instructor D responded to her own questions within two seconds at a higher frequency than the students did and more often than she provided *Minimal Wait-Time, Wait-Time,* and *Extended Wait-Time*. On the other hand, the students in Instructor F's, T's, P's, and C's classes responded within two seconds at a higher frequency than the instructors themselves and more often than *Minimal Wait-Time, Wait-Time,* and *Extended Wait-Time* were provided. Each instructor utilized *Minimal Wait-Time* with the highest frequency when looking at *Minimal Wait-Time, Wait-Time,* and *Extended Wait-Time* in isolation.

Table 30: Wait Time Use by Instructor

	Instructor C	Instructor T	Instructor F	Instructor D	Instructor P
No Wait-Time	69 (28%)	65 (20%)	168 (23%)	108 (38%)	71 (14%)
Minimum Wait-Time	48 (20%)	31 (10%)	66 (9%)	40 (14%)	93 (18%)
Wait-Time	27 (11%)	26 (8%)	41 (6%)	30 (11%)	65 (13%)
Extended Wait-Time	10 (4%)	17 (5%)	33 (5%)	11 (4%)	68 (13%)
Immediate Student Response	91 (37%)	180 (56%)	416 (57%)	92 (33%)	211 (42%)
Total	245 (100%)	319 (99%)	724 (100%)	281 (100%)	508 (100%)

Note: Percent less than 100 due to rounding.

In order to further analyze the instructors' use of wait time, questions coded as *Immediate Student Response* were omitted because it represents something that the students did. When the students responded immediately, there was no way to determine if the instructors would have provided wait time or responded themselves within two seconds. ANOVA was used to investigate the relationship between the number of questions asked and the wait time classification. Based on the results (F=5.40, p=.0092), the average number of questions differs for at least one of the wait time classifications. Further analysis (via a Tukey's multiple comparisons test) revealed the following: (1) on average, instructors asked the same number of questions for which they did not wait for a response or only provided minimal wait-time; (2) on average, instructors asked the same number of questions for which they provided minimal wait-time, wait time, or extended wait-time; (3) on average, the number of questions for which instructors provided no wait time is significantly greater than the number of questions for which they provided minimal wait-time, wait-time or extended wait-time.

Again, due to violation of some ANOVA assumptions, non-parametric procedures were also performed to examine differences in the number of questions posed based on wait time classification. Boxplots of Wilcoxon Rank Sums Scores based on wait time where 1=No Wait-Time, 2=Minimal Wait-Time, 3= Wait-Time, and 4=Extended Wait-Time can be seen in Figure 18.

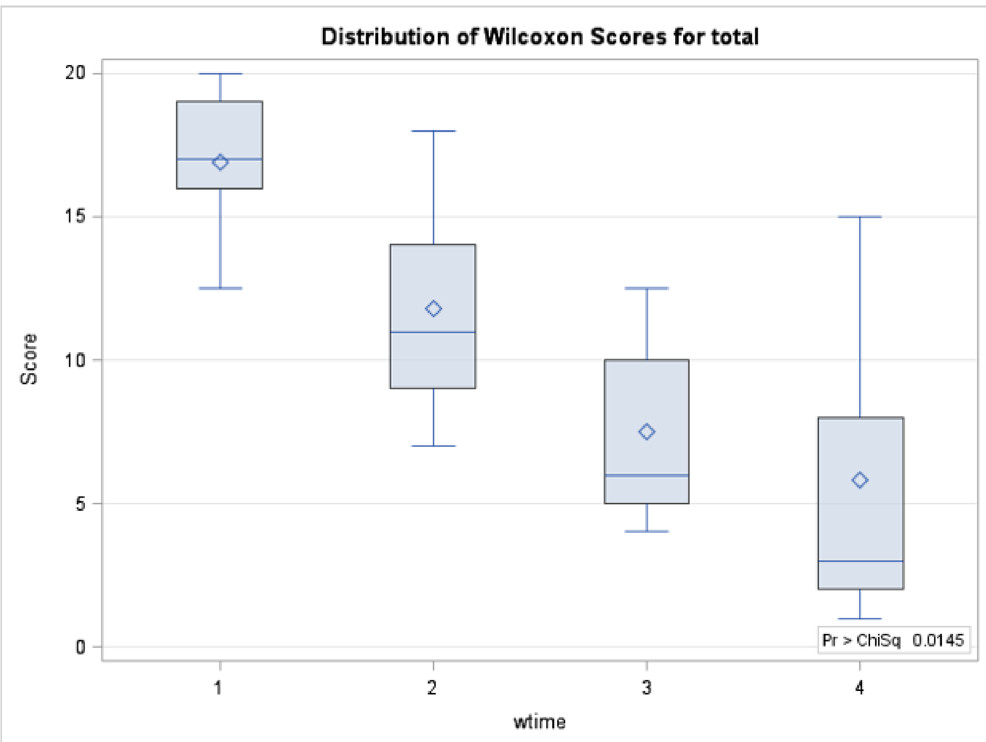

Figure 18: Boxplots of Wilcoxon Rank Sums Scores based on wait time

A Kruskal-Wallis test showed that there was a statistically significant difference among the number of questions posed based on wait time, $\chi^2(2) = 10.54$, p =.0145. A Kruskal-Wallis-based multiple comparisons procedure (Elliott and Hynan, 2011) revealed a significant difference between *No Wait-Time* (Rank Sum=84.50) and *Extended Wait-Time* (Rank Sum=29). As mentioned previously, non-parametric tests are often not as statistically powerful and do not detect differences that parametric procedures, such as ANOVA, detect. In this case, the Kruskal-Wallis test only detected a difference between the wait-time classifications with the highest and lowest median rank scores, respectively.

A chi-squared test was performed to test the independence of wait time and instructor (H_0=wait time and instructor are independent vs. H_a=wait time and instructor are not independent). The results indicated that wait time depends on the instructor ($\chi^2 = 95.7987$, p-value<.0001). Practically, the amount of wait time varies among instructors.

I also combined *Minimal Wait-Time, Wait-Time,* and *Extended Wait-Time* to one broad category "wait time" and looked for emergent themes with just two broad groups of codes, no wait time and wait time. All of the question types were coded as no wait time at some point, yet Instructors C, F, and P had questions types that were never identified as having no wait time (*Extending Thinking, Inserting Terminology, Orienting/Focusing,* and *Requesting Validity*). Common to all three instructors was *Inserting Terminology.* As previously stated, questions followed by no wait time always received an instructor response. All of the instructors in this study with the exception of Instructor F responded with a comment or explanation most frequently. Questions that were followed by a comment or explanation were mainly of the *Checking Understanding* type and could have been answered nonverbally. This included questions like "Does this make sense?" and "Any questions?" Unlike the other instructors, Instructor F's most frequent response was an answer to the question posed. Each instructor was observed answering their own questions, yet Instructor F did so more regularly. Questions that the instructors answered were mainly of the *Gathering Information* type.

Similar to no wait time, all question types were identified as having wait time at some point when looking across instructors. Each instructor did, however, have question types that were not ever followed by wait time. These question types occurred with low frequencies and were either immediately responded to by students or followed by no wait time. Both the students and instructors generally responded to the questions that were given wait time. A student answer occurred with the highest frequency for Instructors P, F, and C although the instructors sometimes answered their own questions. Instructor D also answered some of her own questions, but Instructor T was the exception because he never did this. It was observed that each of the instructors answered their own questions in lower numbers when they provided wait time. An instructor comment or explanation was the most frequent response to both Instructor D's and T's questions with wait time.

The data in Table 31 shows that Instructors T, C, and P were similar in that they used wait time with a higher frequency than they used no wait time which was true for the instructors as a whole, yet the reverse held true for Instructors D and F. Results of one-sample t-tests performed with the data to determine whether there was a significant difference between the percent of questions given no wait time and wait time can also be found in Table 31.

Table 31: Percent of Questions Identified as No Wait Time and Wait Time

	Instructor C	Instructor T	Instructor F	Instructor D	Instructor P	Overall
No wait Time	69 (28.2%)	65 (20.3%)	168 (23.2%)	108 (38.4%)	71 (14.0%)	481 (23.2%)
Wait time	85 (34.7%)	74 (23.2%)	144 (19.9%)	81 (28.8%)	226 (44.5%)	610 (29.4%)
Number of Questions	245	319	724	281	508	2077
t-value	1.287	0.786	1.345	1.977	9.801	3.91
Degrees of freedom	244	318	723	280	507	2076
Two-tailed probability	.199	.432	.176	.049	<.001*	<.001*

Significance is indicated by (*)

The results indicated that Instructor P was the only instructor who displayed a significant difference (p<.001) in his use of wait time compared to no wait time. Comparable results (p<.001) were found when the instructors' questions were viewed as whole.

Classroom Setting

The instructors in this study asked questions during five in-class activities, specifically, lecture-discussion, individual work, small group work, whole group discussion, and student presentations. Whole group discussion was the only in-class activity observed across all instructors, yet it was not utilized the same. Instructor P's entire class was structured around whole group discussion. The class worked on both student and instructor selected problems in a whole group setting throughout the duration of all observed class sessions, as such all of the instructor's questions were posed in this particular classroom setting. As previously noted, he asked all question types minus *Linking/Applying*.

Whole group discussion was also observed daily in Instructor T's and Instructor C's classes during this study. Both instructors asked most of their questions during this in-class activity. Additionally, they provided wait time more often than not when engaged in whole group discussion. Instructor T asked a larger range of questions types than Instructor C during whole group discussion, but both instructors gave their students opportunities to think deeply about mathematics with their use of *Probing* and *Meaning/Relationships* questions. The instructors' use of whole group discussion was also different in that Instructor T's students were engaged in this in-class activity after spending time working in small groups, and Instructor C's students worked individually prior to whole group discussion.

Instructor D utilized whole group discussion on most of the observed days as well, yet unlike the instructors previously mentioned this was not the in-class activity where she asked the most questions nor was it where she asked the most question types. Instructor D's use of wait time during this in-class activity was also different than Instructors P, T, and C because she used no wait time at a higher frequency than wait time. Similar to the other instructors, she gave students an opportunity to think deeply with her use of *Probing* questions during whole group discussion. This in-class activity was in reference to homework, individual work, and small group work.

Whole group discussion occurred on two days after a quiz in Instructor F's observed classes. *Gathering Information* was the main question type used. Thanks to one *Probing* question this in-class activity also had evidence of students being asked to think deeper in Instructor F's class. Little time was devoted to this in-class activity and, like Instructor D, this was not where the highest number of questions were posed.

Lecture-discussion was the dominant in-class activity for Instructors D and F, meaning the bulk of their daily class time was spent in this way, and most of their questions were asked during this in-class activity. Both instructors' lecture-discussions entailed new material as well as review. Similarly, they both provided wait time more often than not when they reviewed. Instructor C also used lecture-discussion daily, but it was limited to the first half of class. No wait time was more common than wait time during Instructor C's lecture-discussions. The same was true for Instructors D and F when they worked on new material. Instructors D and F had evidence of all 11 question types during their lecture-discussions. Instructor C asked seven of those question types, namely, *Gathering Information, Checking Understanding, Requesting Validity, Probing, Extending Thinking, Inserting Terminology,* and *Context. Gathering Information* questions were asked more than any other question type during this particular in-class activity regardless of who the instructor was.

In addition to lecture-discussion and whole group discussion, Instructors C and D had their students work individually. This was a daily occurrence for Instructor C's observed class sessions, yet the same was not true for Instructor D. There was also minimal evidence of this in-class activity on one day of Instructor T's observed class. Instructors T and D asked very few questions as students worked individually; two and four respectively. *Checking Understanding* and *Requesting Validity* questions were asked by both instructors, however, Instructor D also asked a *Gathering Information* question. Instructor C asked questions of these types as well as *Meaning/Relationships, Probing*, and *Inserting Terminology* while students worked individually. He too asked fewer questions while students worked individually.

Small group work was another in-class activity that Instructor's D and T had in common. Students were engaged in small group work twice a day in Instructor T's observed classes, whereas it was an occasional in-class activity for Instructor D. Instructor T asked numerous questions during small group work, but they differed in frequency, type, and wait time depending on if the students were working on a warm-up or a challenge problem. He asked *Checking Understanding, Gathering Information, Requesting Validity, Probing, Orienting/Focusing* and *Generating Discussion* questions during small group work. Instructor D, on the other hand, asked very few questions as students worked in small groups.

Chapter Summary

This chapter detailed findings of the question types, the wait time used, and the classroom setting(s) utilized by five undergraduate Calculus I instructors. The instructors in this study used their questions in distinct ways, but there were commonalities drawn across them. In brief, each of them asked a variety of questions including *Gathering Information, Requesting Validity, Checking Understanding, Probing, Meaning/Relationships, Extending Thinking,* and *Inserting Terminology* and used *Gathering Information* with the highest frequency. Their students often answered their questions, and they used wait time as well as no wait time with no reservation for a particular type of question. They followed their questions with comments, explanations, other questions, and sometimes answered their own questions. Additionally, they were all observed using whole group discussion. Although these broad themes can be drawn, the instructors asked the question types in varying frequencies, utilized the in-class activities differently, and with different wait times.

Instructors D and F were unique in that they utilized all 11 question types and did so during lecture-discussion. Instructor P was the only instructor to utilize one in-class activity throughout the duration of the observed class sessions, and the only one who used wait time significantly more than no wait time. Instructor T was the only instructor that did not answer his questions when they were followed by wait time or ask *Extending Thinking* questions, and the only one who engaged his students in presenting their work. Instructor C utilized the least question types.

In the next chapter, the findings will be discussed as they relate to answering the research questions. Additionally, connections to literature will be made and implications for the field, limitations of the study, and potential future research will be addressed.

CHAPTER 5: DISCUSSION AND CONCLUSIONS

The purpose of this study was to provide a description of how undergraduate Calculus I instructors use their questions during discussions about mathematics. In order to accomplish this goal, classroom observations were conducted to capture the discourse that ensued in five Calculus I classes. A conceptual framework informed by literature was created with emphasis on the verbal questions instructors ask because of their in-the-moment value for both instructors and students (Martino & Maher, 1999; Roach et al., 2010). The framework recognizes that the mathematical questions instructors ask are followed by varying wait time (Rowe, 1986) and there are many paths discourse can take. It also acknowledges that instructors' questions are embedded within in-class activities that also contribute to understanding how they are used. The goal was to answer the overarching research question: *How are mathematics instructors using their mathematical questions in undergraduate Calculus I classes?* Rich descriptions of the case instructors' questioning as well as a cross-case analysis were provided in the previous chapter. This chapter utilizes the findings detailed in the previous chapter to answer the research question specifically organized around the sub-questions.

a. What types of questions are asked by Calculus I instructors?

b. In what ways do instructors utilize wait time with their mathematical questions?

c. In what class setting(s) are the various question types and wait time observed?

It concludes with a discussion of the limitations, implications of this study, a presentation of the study's and suggestions for future research.

Instructors' Questioning in Calculus I

This section summarizes the findings as they relate to each of the research questions posed and makes connections to literature discussed in Chapter Two. Each sub-question is answered, and the section concludes with a summary as it relates to the overarching question.

Question a: What types of questions are asked by Calculus I instructors?

All nine question types described in Boaler and Brodie's (2004) work were identified in this study indicating that Calculus I instructors ask the same kinds of questions observed in secondary mathematics classes. Two additional question types (*Requesting Validity* and *Checking Understanding)* were utilized that have been seen in more recent research conducted in undergraduate and secondary mathematics classes (Mesa, 2010; Mills, 2013; Stolk, 2013). This included questions like "Is that right?" and "Any questions?" respectively. Stolk (2013) referred to questions of this type as *Analyzing the Validity* and *Analyzing Personal Understanding* as subcategories of questions asked by an undergraduate mathematics instructor in a conceptually oriented classroom. Questions that focused on validity asked students to form an opinion of the legitimacy of an argument, and personal understanding was about having students self-reflect on their understanding. Mills (2013) described questions similar to this as *Choice* questions as dictating that students agree or disagree with a given statement and *Comprehension* questions as checking understanding. Mesa (2010) discussed *Sentence-Right?* and *Rhetorical* questions which respectively include examples like "y equals one. Right?" and "Does that make sense?" as questions that instructors did not intend for students to answer. Boaler and Brodie (2004) may have taken a view similar to Mesa (2010), hence the absence of the added question types. I did not want to assume whether or not the instructors intended for the students to answer, nor did I have data to indicate intentions. Furthermore, I found that students sometimes answered these questions, and in instances wait time was provided suggesting that the instructors may have expected students to respond even if it was nonverbally. I adapted Boaler and Brodie's (2004) categories of questions such that *Requesting Validity* and *Checking Understanding* were included as mathematical questions which resulted in 11 question types. Approximately 20% of the instructors' questions were of the *Requesting Validity* and *Checking Understanding* types. Similar results were found in both Stolk's (2013) and Mills' (2010) work.

Seven types (*Gathering Information, Requesting Validity, Checking Understanding, Probing, Meaning/Relationships, Inserting Terminology,* and *Extending Thinking*) were common to each of the instructors' mathematical questions showing that each of them asked a variety of questions. This is different than Boaler and Brodie (2004) who found that there was only one question type, *Gathering Information,* common to the six secondary teachers in their study. Like Boaler and Brodie (2004), I found that *Gathering Information* questions made up the highest percentage of each instructors' questions (between 30-75%), and *Extending Thinking, Linking/Applying, Orienting/Focusing,* and *Inserting Terminology* questions were asked sparingly (around 2-4% together). Statistical analysis performed after reorganizing the question types, as described in the cross-case analysis, indicated that the instructors as a whole asked *Gathering Information* questions significantly more than the other question types based on ANOVA, but only more than *Meaning/Relationships* and the *Collapsed Category* when non-parametric methods were used.

The instructors asked *Gathering Information, Probing,* and *Generating Discussion* questions in ways that suggests subtypes within the categories. Many of the *Gathering Information* questions were posed in ways that support the subcategories described in Stolk's (2013) *Accessing Relevant Information* category found in Figure 19.

Form of the Answer [Code]	Description	Example Questions
Accessing a Past Class Experience/ Past Answer [AccessExperience]	Recalling and sharing past class experiences or answers that have not been reflected on yet	"Did anyone roll a 7?" "What did you get for number 3?" "Who found that it was hard to roll a 2?"
Accessing a Numeric or Computational Result [AccessAnswer]	Identifying or performing a computation	"What is 2/3 of the group?" "What is 50% of that number?"
Accessing a Past Idea [AccessIdea]	Bringing up old ideas from class	"What did we talk about yesterday?" "How did we define [that idea] before?" "What were the elements to a good explanation that we talked about last time?"
Accessing Context [AccessContext]	Talking about real world contexts, personal experiences, or knowledge outside of mathematics that can/will be used in the mathematics problem	"How many of you have played Settlers of Cataan?" "How many brothers and sisters do you all have?" "How do meteorologists decide the chance of rain?"
Accessing a Past Procedure [AccessProcedure]	Identifying a known process	"How do you do long division?" "What do you do to find what it's equal to?"

Figure 19: Stolk's (2013) Accessing Relevant Information category of questions

In this study, Stolk's (2013) *Accessing Context* subcategory represented its own question type, *Establishing Context*, as described in Boaler and Boaler (2004). I do, however, agree that contextual information is one of many kinds of information that can be gathered with instructors' questions.

In addition to supporting Stolk's (2013) Accessing Relevant Information category, the instructors in this study asked *Gathering Information* questions that fall in line with some of Dong et al.'s (2015)subcategories of initiation questions. Dong et al. (2015) presented *Review* and *Result/Product* questions which focus on previously learned material and the results of mathematical operations or answers. Both subcategories align with questions discussed in Boaler and Brodie's (2004) and Stolk's (2013) work, yet like Dong and colleagues I also found that the instructors asked students to *extract* information from text descriptions, figures, etcetera. As a whole, the questions posed to *Gather Information* collected a range of information and presented students with opportunities to engage in discourse.

Similar to the *Gathering Information* question type, the instructors asked *Probing* questions that asked for different information. Hähkiöniemi (2013) identified seven types of probing questions (*Probing method, Probing reasoning, Probing cause, Probing meaning, Probing argument, Probing extension,* and *Unfocused*), four of which were put to use by the instructors in this study. Each of the instructors were observed asking their students to explain how they got an answer or how they did something representing *Probing method* questions. They also asked their students to justify claims or explain how they knew something in alignment with what Ḧahkiöniemi (2013) considers *Probing argument*. Many of the instructors asked follow up questions that asked students to explain the meaning of something (i.e., *Probing meaning*). There was also evidence of *Probing cause* questions that asked students why they did something.

Subcategories of follow-up questions provided by Dong et al. (2015) were also identified in the *Probing* questions asked by the instructors in this study. Dong et al. (2015) described *Clarification* questions as those that asked students to provide more detail about their answers which is like Hähkiöniemi's (2013) *Probing method* subcategory. *Justification* questions similar to Hähkiöniemiʹs (2013) *Probing argument* subcategory were detailed as well. Each of the instructors probed their students to provide additional information that was not specific. These questions were identified in Dong et al.'s (2015) work as the *Elaboration* subcategory. An example from the data occurred as a student explained that they were optimizing the area in a problem. They were probed for more information when the instructor said, "The area of what?" The *Probing* questions posed followed-up on a variety of things, however, they all presented opportunities for students to further explain their thinking.

Boaler and Brodie (2004) described *Generating Discussion* questions as soliciting responses from other students in the class. Many of the instructors who used this question type did so in this way. This was seen when the instructors asked if anyone had anything to add to an explanation and for alternative ways to do the same problem. This type of question appears in other categorizations of questions as well, for example Dong et al. (2015) called these *Supplement* questions in their follow-up category.

Like *Probing* questions, most *Generating Discussion* questions gave students an opportunity to explain their thinking. *Generating Discussion* questions are different than *Probing* questions in that they do not follow-up on students' thinking. These questions encouraged students to share their thoughts and ideas and were at times about solving problems or what things could mean. Instructors D and P *Generated Discussion* in an additional way that involved asking students what they wanted to do during class. This particular kind of *Generating Discussion* is not about students explaining mathematics, but the questions elicit their thoughts nonetheless. Questions coded as *Generating Discussion* were identified as such because of their potential to spark discussion; they did not necessarily produce discussion.

The questions asked by the Calculus I instructors in this study demonstrated that it was common to utilize the same types of questions when teaching mathematics. Be that as it may, the same types of questions did not yield similar teaching practice in regard to questioning. Statistical analysis showed that the question type (excluding *Generating* Discussion) depends on the instructor indicating that there are question types that we can anticipate that one instructor would ask over another. Aside from that, the instructors engaged their students differently due in part to their use of wait time and their classroom setting discussed later.

Question b. How do instructors utilize wait time during their discussions about mathematics?

Broadly, each of the case instructors were observed providing both wait time and no wait time when their students did not respond to questions within two seconds. Three specific kinds of wait time (*Minimal Wait-Time*, *Wait-Time*, and *Extended Wait-Time)* were utilized with an array of question types, including *Gathering Information,* showing that the instructors did not reserve wait time for questions that required deeper thought. Although all of the instructors utilized each wait time category, statistical analysis showed that the wait time category depends on the instructor. This suggests that one instructor may be more likely than another to utilize a particular wait time category. As a whole the instructors used wait time significantly more than no wait time ($p<.001$) suggestive of an average wait time higher than the one second average that has been reported in literature (Aizikovitsh-Udi & Star, 2011; Kwit, 2012; McComas & Abraham, 2004; Rowe, 1986). Instructors P, T, and C used wait time at a higher frequency than they did no wait time, however, Instructor P was the only one who did so in a way that produced a significant difference in the percents of each ($p<.001$). Instructors D and F put no wait time to use at a higher frequency compared to wait time, yet again there was not a significant difference. Failure to find a significant difference between the percentage of questions followed by wait time and those not given wait time for all five instructors suggests many students are not regularly getting ample time to think and formulate answers. As a result, attention to wait time is an area that needs further study.

Wait times of various lengths have been recommended by researchers and were observed in this study. The wait times that followed the instructors' questions ranged from less than one second on the low end to around one minute on the high end. Each of the instructors utilized the low end, yet the high end was exclusive to Instructors C and P. Approximately 40 to 60% of the instructors' questions recognized as having wait time entailed pauses higher than three seconds. These times exceed the 2.7 seconds "threshold value" identified by Rowe (1986). Further, many of these wait times were around the five second mark described by Tobin (1987) as central to seeing positive changes in discourse. Most of the instructors utilized pauses longer than 10 seconds in alignment with what von Renesse and Ecke (2014) considered exemplary wait time in mathematics classes. The instructors used *No Wait-Time* significantly more than *Extended Wait-Time* as one would expect because of the difficulties described with maintaining longer wait times.

The pauses behind questions showed that wait time was in fact used, but there was little indication that wait time was being consciously used by the instructors. For example, there were many instances of wait time that can be attributed to instructors doing other things (e.g., looking at notes, looking in the textbook, and writing). This was noted as I documented the start and stop times needed to code wait time. Instructors T and P were the only instructors who made statements that let their students know that they were actually pausing for the purpose of giving them time to think about or articulate an answer to the questions posed.

Questions followed by pauses less than two seconds have been described as those that instructors do not expect students to answer (Mesa, 2010), yet I found that this was not always the case. In instances, the instructors made a comment or gave an explanation before giving students a chance to answer the question. Heinze and Erhard (2006) pointed out that instructors often do this and it produces a gap in wait time. Instructors P and T further demonstrated that at times they expected their students to answer questions followed by no wait time when they acknowledged students by name within two seconds to answer the questions. It can be gathered that the responses that follow the instructors' questions may lend themselves to establishing if a student answer was expected.

Each of the instructors were observed quickly answering their own questions indicating that there were some questions that they may not have intended for their students to answer. The instructors answered between 12% and 45% of the questions given no wait time. Instructor T did this the least answering around 12% of his questions within two seconds. On the other hand, Instructor F did this most often as demonstrated by her answering 45% of the questions that she asked and gave no wait time. The questions the instructors answered were generally of the *Gathering Information* type representing questions that the students could answer based on prior knowledge. There was, however, evidence of the instructors answering question from other types.

The instructors were observed responding to questions followed by both wait time and no wait time with comments/explanations, answers, and at times other questions. As a whole, the instructors answers their questions less often when they used wait time. This finding aligns with literature which has described one impact of wait time as instructors being less tempted to answer their own questions (Kwit, 2012; Rowe, 1986; Tobin; 1987). Instructor T represents an exemplar case in that he did not answer any of his questions followed by wait time. The students were observed answering questions with wait time in high numbers in Instructors C's, F's, and P's classes, yet the same was not true for Instructors D and T. Again, Instructors T and P responded uniquely by acknowledging students by name to answer questions which shows their focus on engaging students with the questions posed.

Question c: In what classroom setting(s) are the various questions types observed?

The Calculus I instructors in this study were observed asking questions during five in-class activities (lecture-discussion, individual work, small group work, whole group discussion, and student presentations). Most of the instructors in this study demonstrated variation in the in-class activities used, but there was generally a dominant in-class activity. Lecture-discussion was the dominant in-activity for Instructors C, D, and F, whereas whole group discussion was dominant for Instructors P and T. These particular in-class activities were identified as dominant because they were the in-class activities that the instructors devoted the most class time to. Each of the instructors used at least one other in-class activity, but I found that their dominant in-class activity was also the place that their questions displayed the most variation by type.

A range of question types were observed in each in-class activity, yet Instructor T was the only instructor that was observed using the same question type during a particular in-class activity each day. The instructor only asked *Checking Understanding* questions as the students worked in small groups on their warm-up. He asked a wider variety of questions when students worked in small groups on their challenge problem compared to when they worked on the warm up. There was more time spent working on the challenge problem. This finding in conjunction with the one above about questioning within the dominant in-class activity suggests that time spent on an in-class activity may be connected to the range of question types posed during an in-activity opposed to the in-class activity itself.

Overarching research question: How are mathematics instructors using their mathematical questions in undergraduate Calculus I classes?

Question types, wait time, and classroom setting(s) (i.e. in class activities) work together to detail the ways Undergraduate Calculus I instructors use their mathematical questions. The instructors are using their mathematical questions in the ways described by the question types used to categorize them (Boaler & Brodie, 2004; Mesa, 2010; Mills, 2013; Stolk, 2013), and in instances in ways that support subcategories of question types. All of the question types were not put to use by the instructors individually, yet there was evidence that they asked questions identified in literature as conducive to getting students to explain and engage in understanding mathematics (Hähkiöniemi, 2013; Stein & Smith, 2010; Stolk, 2013). Two instructors gave attention to each of the question types described in this way, which includes all 11 types with the exception of *Gathering Information, Checking Understanding,* and *Requesting Validity* (Stolk, 2013). This finding was of interest because both of the instructors taught large classes compared to the others and did so in classroom settings that were primarily lecture-discussion. They spent a lot of time explaining concepts and problems to students, as such I did not anticipate that they would ask the same types of questions that were observed in more student-centered classes. It could be said based on question type alone that the instructors' questioning was similar in that they generally asked the same types of questions, yet I gathered that in this study the types of questions asked did not distinguish teacher-centered classes from student-centered classes as in Boaler and Brodie's (2004) work. Interestingly, the instructors' teaching practice was not the same which suggests that while it is important to have a means of categorizing instructors' questions, doing so does not detail the teaching practice of questioning itself. Question types do, however, have their place in understanding instructors' questioning practice. First, they tell us what the instructors are asking of their students with the questions they pose. Additionally, they can be used to identify questioning patterns and any associated interaction patterns which attribute to detailing what instructors do during the practice of questioning.

This study looked beyond question types and gave explicit attention to wait time to help establish the ways instructors use their questions. Wait time is not always a part of research on teacher questioning (Boaler & Brodie, 2004; Mills, 2013; Roach et al., 2013), but it was included in this study because the question types themselves only determine what the instructors seek to do with the questions. Wait time sheds light on the opportunities students have to answer them. I found that the instructors used wait time behind their questions and did not utilize it for a particular question type, however, differences in the amount of wait time the instructors provided and the responses they offered led to different opportunities for students to respond to the questions posed. The idea of wait time versus no wait time is of importance because it represents the time students are given to think and formulate responses to the questions posed, yet I posit that the responses that ensue are necessary for understanding wait time as a part of instructors' questioning practice. For example, an instructor who regularly answers their own questions exhibits a different practice than an instructor who answers a minimal number of their own questions, or one who is the first to respond but gives students subsequent opportunities to answer their question(s).

In Boaler and Brodie's (2004) work teacher questioning was recognized as one of many mutually exclusive in-class activities that students were engaged in during mathematics classes. In this study, teacher questioning was recognized as a teaching practice that can occur within a plethora of in-class activities. This was deemed essential to understanding instructors' questioning because treating it as an in-class activity neglects questions that may be posed during other in-class activities. Boaler and Brodie (2004) acknowledged that there were questions posed in other activities that were not captured in their analysis and stated that they were later analyzed. This study shows that in-class activities not only help detail the classroom setting, but they contribute to understanding instructors' questioning.

The instructors asked the largest range of questions during the in-class activity they spent the most time on, as such the types of questions posed were not so much about the in-class activity itself. For example, an instructor who utilizes majority lecture-discussion may not ask as many question types during whole group discussion; not because they limit questions during whole group discussion, but because they do not spend as much time engaged in that activity. At the same time, the questions types included in their lecture-discussion may be the same types as those seen in someone else's whole group discussion. The instructors did use their questioning in ways that suggest the in-class activity they are engaged in is connected to their use of wait time.

Limitations

Efforts have been taken to minimize the limitations of this study, yet like all other research this study presents limitations. First, the dataset was a lot to manage for a novice researcher. My experiences as a student research assistant proved to be helpful with collecting, organizing, and analyzing data. Further, the expertise of my dissertation committee was sought to offset this limitation. The method used to calculate wait time presents another limitation. Wait time was calculated using Atlas.ti and a mouse so their accuracy is dependent on an individual's reaction time. Yet I took measures to reduce reaction time and acknowledged that they may be approximately ±.2 seconds off. With this ±.2 difference, I was still able to code according to the norms in literature.. Lastly, the instructors' perceptions of their questioning was not captured and potentially presents a limitation.

Implications

This study has provided insight into the ways undergraduate instructors are using their mathematical questions in Calculus I. Questioning has been identified as an important and under examined teaching practice at the undergraduate level (Roach et al., 2010; Speer, Smith, & Horvath, 2010), as such this study contributes to the field of research in undergraduate mathematics education by presenting empirical research. The results of this study have value for those in charge of designing the class structure, undergraduate mathematics instructors, professional development designers and/or providers and future researchers.

There are implications for the individuals in charge of designing the class structure of Calculus I courses. It is known that some universities have a student population that does not allow for small class sizes and all classroom layouts are not conducive to small group work. These environments are usually tagged heavily lecture and lacking student engagement. While lecture is not ideal, there were two classes in this study that utilized lecture regularly as part of lecture-discussion that demonstrated that instructor questioning can be a useful mechanism for engaging students in discourse and learning mathematics beyond facts and procedures in larger classes. Instructors in larger classes can also give students an opportunity to work with peers by having them turn and talk to a partner about problems as seen with Instructor D. This can help those types of class structures improve their students' engagement in lessons.

Instructor T represented an exemplar case. He had as many students as Instructor F, but his classroom layout was not the typical rows of tables and chairs. Students were organized in pairs at computers which instantly made room for student interactions. The instructor did not spend time lecturing at the front of the room and spent minimal time explaining. Students worked with their peers on two problems a day, and were asked a range of questions that kept them engaged during whole group discussions about mathematics.

For undergraduate mathematics instructors, the conceptual framework and the coding scheme represent valuable tools for reflecting on this particular teaching practice. The conceptual framework draws attention to wait time which McComas and Abraham (2004) pointed out is often overlooked when questioning takes place. It also acknowledges that there are many paths that discourse can take after questions have been asked and may itself be a potential instrument for mapping both questioning and interaction patterns. Lastly, it considers the fact that instructors ask questions during in-class activities which may not always be the same.

The coding scheme was developed and adapted based on literature and the classroom observation data, yet it was guided by the conceptual framework. This study provides evidence that the questions posed by undergraduate Calculus I instructors during discussions about mathematics can be categorized by the nine question types described in Boaler and Brodie's (2004) work with secondary mathematics teachers and two additional types presented in more recent research (Mesa; 2010; Mills, 2013; Stolk, 2013). The coding scheme entails detailed categories of questions adapted from the aforementioned literature that can be used as a tool for learning about the types of questions they ask and reflecting on the ones they use. Further, it includes a system for coding wait time that separates wait time into categories that can be useful for instructors' general understanding of wait time.

Additional codes that draw attention to who responds to instructors' questions are of equal importance because instructors may not realize how often they answer their own questions or interrupt students' thoughts by commenting or explaining. The conceptual framework, coding scheme, and data from this study have value for professional development (PD) designers and/or providers. Each can be used as foundational materials for designing and implementing PD that focuses on instructors' verbal questioning in undergraduate mathematics. PD developers can design activities that would allow the instructors to analyze data using the codes around both question types and wait time. Instructors could analyze their own questions or those of others using video data which has been deemed an effective PD tool (Borko, Jacobs, Eiteljorg, & Pittman, 2008; Van Es & Sherin, 2008).The data in this study would serve as fruitful artifacts of others' questioning since it contains more than 2000 questions, evidence of all 11 question types, and each category of wait time. Video data will give instructors a real-life feel for wait time by allowing them to count the seconds in the pauses and capture who is responding. In addition to analyzing data, instructors can edit transcripts using alternative questions that would engage students deeper than what is presented. They can also use transcripts to determine the different ways that the same question type can be used (e.g., what types of information can instructors gather with *Gathering Information* questions).

It is believed that a professional development of this nature would be of importance because it would encourage instructors to reflect on their questioning and potentially produce better practice. This is something instructors should do for multiple reasons but two are addressed here. First, the questions that instructors model for students are the ones they tend to ask themselves and each other when working on problems (Stolk, 2013). Some students may be pre-service teachers and others may also end up teaching as graduate assistants, so instructors' questions may have a pedagogical influence. Second, all students should be engaged with questions that afford them opportunities to conceptually understand mathematics.

As stated earlier, there are implications for future researchers presented by this study. First this work was conducted using a tool that has mainly been used at the K-12 level to discuss mathematics teachers' questions. I have found the tool to be effective for establishing the types of questions undergraduate Calculus I instructors ask and would suggest that future researchers utilize the tool to explore questioning in the same course or other undergraduate courses. The conceptual framework and the coding scheme as a whole would be beneficial for further detailing the ways instructors are using their questions. We need more research in this area because it is not a teaching practice that is fully understood, and a further understanding of instructors' questioning will help with gauging its impact on students.

An additional implication for researchers can be found in the coding of wait time. For this study, wait time was defined as the pause between the instructor's question and the initial utterance of the following response. There were instances of both students and instructors being the first to respond to the questions posed, yet the response was not always the answer to the question. For example, it could have been an instructor comment, explanation, or acknowledgement. Heinze and Erhard (2006) argued that such responses create gaps in the initial wait time and defined Wait Time 3 to compensate for the time the instructor may pause after their subsequent response. Consider when the instructor poses a question and acknowledges a student by name to respond. If the pause was less than two seconds, it was coded as no wait time in this study because the instructor was the first to speak after the question was posed. Further, I viewed it as the instructor no longer waiting as a student had been called on to respond. According to Heinze and Erhard (2006), the pause between the acknowledgment and the student's response would be added to what was defined as wait time in this study. The results regarding wait time may be impacted by the way wait time is defined and coded, and it presents an area for future research.

Future Research

This study focused on the types of questions undergraduate Calculus I instructors asked, the wait time they provided, and the classroom setting(s) where their questions were asked for the purpose of shedding light on the ways undergraduate instructors use their mathematical questions. Teacher questioning has been identified as an under examined teaching practice at the undergraduate level (Speer, Smith, & Horvath, 2010), and this study addressed this issue by providing rich descriptions based on empirical data collected in its natural environment. Ideas for future research involve detailing undergraduate Calculus I instructors' teaching practice in a more meticulous fashion. This can include considering data collection differences (e.g. observing over a longer period of time, focusing in one instructor).

The research can also be extended by analyzing all the discourse observed with the exception of student-to-student interactions during small group work so that explaining, interaction patterns, student participation, and other factors related to instructors' questioning can be discussed. This is necessary because there were differences in the instructors' teaching practice that did not show up in this study because of the focus of the data analysis. For example, student-to-student interactions that resulted from Instructor T's questioning during student presentations were not addressed. Additionally, the questions that instructors asked were often embedded in explanations that also contributed to describing the ways instructors used their mathematical questions. Future research should tend to interaction patterns as well as questioning patterns. The conceptual framework developed in this study offers a means of capturing both the interaction and questioning patterns utilized during instructors' questioning because it can be used as a mapping tool. Research of this nature would provide insight on the sequencing of question types and help with detailing how instructors are engaging students with their mathematical questions.

An additional direction for future research involves introducing an undergraduate instructor to the adapted categories of questions and having them reflect on their own questions. I am interested in seeing how instructors analyze their own questions. This will provide information on their intentions when asking questions, but would require collecting additional data so that the instructors can recall why they asked particular questions. It would also be interesting to draw instructors' attention to their explaining. Some of the instructors in this study did a massive amount of explaining, and I was left wondering if they even realized that they talked in such large chunks.

Another area of interest would be to take a further look at question types. The data supports the idea that there are subtypes within question types. The subtypes recognized in the *Gathering Information* and *Probing* types suggest that there may be even be different levels within question types. Consider *Probing* questions that ask students about a method versus those that ask for justification. Students are aware of what they did, so it should not require much thought for them to share that with the class. On the other hand, they may not have given thought to why they did it. How are instructors using these subtypes? Do they use one subtype more than another? I am left wondering what other question types and/or subtypes may have shown up if I had observed for a longer period of time or if I had observed more instructors for a shorter period of time. This is an area worth exploring because it could add to understanding the types of questions instructors ask and how they use them.

A more fine-tuned look at wait time is also of interest. Although it was established that the Calculus I instructors in this study did not use wait time for particular types of questions, further analysis could determine if wait time was given more or less for particular types of questions. Only two instructors verbalized that they were actually waiting for students to respond, but each of the instructors utilized wait time. There were also times that wait time could be attributed to the other things, so in future research I would look for potential nonverbal cues that instructors may use to indicate that they were waiting intentionally.

Conclusion

Instructor questioning is one of the most important teacher practices because of its ability to impact student engagement and understanding (Henningsen & Smith, 1997; Kazemi & Stipek, 2001). It is a practice that has not been heavily detailed at the undergraduate level although its importance has been recognized. This study presents empirical evidence that question types informed by research in K-12 mathematics classrooms (Boaler and Brodie, 2004) are appropriate for studying questioning at the undergraduate level. It confirmed that instructors often ask *Gathering Information* questions, but showed that they are asking them in conjunction with noticeable numbers of questions that engage students in sharing their thoughts and explaining. The instructors' use of wait time challenged the notion that instructors typically wait approximately one second after asking questions, yet it confirmed the fact that instructors often display difficulty maintaining wait times of three to five seconds. Instructors ask questions in Calculus I classes of different sizes in environments that may not lend themselves to in-class activities common to student-centered environments, but I contend that the appropriate mix of questions can still produce valuable interactions when students are given an opportunity to formulate answers and respond to questions.

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APPENDICES

Appendix A: Initial Contact Email for Department Chairs

Dear Dr. XXX,

My name is Tracy Foote White, and I am a doctoral candidate in the department of Science, Technology, Engineering, and Mathematics Education at North Carolina State University. The purpose of this email is to solicit your assistance with identifying potential participants for my dissertation study. My dissertation is a qualitative multi-case study that aims to describe and understand teacher-student interactions in Calculus I classes. I am contacting the mathematics department chairs at local universities eager that you all will help.

I am requesting that you refer two-three Calculus I instructors from your department who think would be interested and willing to be participants. Each instructor will be asked to participate in an interview. If they selected for the study there will be six consecutive hours of classroom observations. Please send your recommendations via email to tsfoote@ncsu.edu.

If you have any questions or require clarifications, please contact me at xxx.xxx.xxxx or tsfoote@ncsu.edu. Dr. Allison McCulloch, chair of my dissertation committee, may be contacted at xxx.xxx.xxxx.

Thank you for your cooperation,

Tracy Foote White
Doctoral Candidate
Department of STEM Education
College of Education
NC State University

Appendix B: Initial Contact Email for Instructors

Dear XXXX,

My name is Tracy Foote White, and I am a doctoral candidate in the department of Science, Technology, Engineering, and Mathematics Education at North Carolina State University. I asked your department head for a recommendation of a good Calculus I instructor who may be willing to participate in a small research study, and she/he suggested you.

My dissertation is a qualitative multi-case study that aims to describe and understand teaching practices in undergraduate classes. The study is not intended to evaluate or judge your teaching in any way. Participation will require an interview and approximately six consecutive hours of classroom observations.

If you are interested in volunteering for this study or need clarifications, please contact me at tsfoote@ncsu.edu.

Thanks for your time,

Tracy Foote White
Doctoral Candidate
Department of STEM Education
College of Education
NC State University

Appendix C: Interview Protocol

Say: Thank you for agreeing to participate in this interview. I would like to talk a little about your background and teaching practice. I would like to record this interview so that I can focus on what you are saying. The audio-recorder will be used to capture your responses because I will not be able to document everything by hand. Is that okay? I will be taking minimal notes during this interview. Do you have any questions before we begin?

1. Tell me a little about yourself and how you decided to teach mathematics.

   -Is it what you envisioned?

2. Describe what your typical Calculus I class-session looks like.

   -What are you doing?

   -What is a typical student doing?

3. Describe in detail the most recent Calculus I lesson that you have taught?

   -What were you doing?

   -What was a typical student doing?

4. When you reflect at the end of a class, how do you know whether or not your students have learned what you were hoping they would?

5. What are some important skills you would like your students to have upon leaving your class?

Say: Is there anything you would like to add that was not asked? Thanks for your time. I am looking forward to observing your classes. Do you mind if we take a few moments to set up observation dates and times?

Appendix D: Consent Forms

Instructor Consent Form

North Carolina State University

INFORMED CONSENT FORM for RESEARCH

Title of Study: Teacher questioning in undergraduate mathematics: A multi-case study

Principal Investigator: Tracy Foote White

What are some general things you should know about research studies?
You are being asked to take part in a research study. Your participation in this study is voluntary. You have the right to be a part of this study, to choose not to participate or to stop participating at any time without penalty. The purpose of research studies is to gain a better understanding of a certain topic or issue. You are not guaranteed any personal benefits from being in a study. Research studies also may pose risks to those that participate. In this consent form you will find specific details about the research in which you are being asked to participate. If you do not understand something in this form it is your right to ask the researcher for clarification or more information. A copy of this consent form will be provided to you. If at any time you have questions about your participation, do not hesitate to contact the researcher named above.

What is the purpose of this study? The purpose of this study is extend what is known about teacher practices in undergraduate mathematics.

What will happen if you take part in the study?
If you agree to participate in this study, you will be asked to participate in a list of activities:

- **Interview**: 30-45 minutes. This interview will consist of questions that allow the instructor to discuss their background and teaching practice. It will be conducted prior to the collection of any classroom observation data. The interview will take place in a location convenient for the instructor; perhaps at their university. The interview will be audio-recorded.

- **Classroom Observations**: Each observation will last the duration of the class session. Observations will be conducted during the spring 2015 semester for approximately six consecutive class hours. Classroom observations will be video recorded with three video-recorders. One will follow the instructor, one will be located in a place that captures the full classroom if possible, and the other will capture anything that is displayed or written on the board. Recording will take place from the least obtrusive locations in the classroom. The instructor will also wear a lapel microphone connected to a digital audio- and video-recorder. The observations will take place in the location that classes are held.

Risks

There are no physical or emotional risks associated with participation in this study.

Benefits
I will share the study findings with the instructors which will include a framework that they can use to reflect on their own practice. Instructors can also share the framework with their departments. Documenting interactions

at this level of mathematics education will inform an emergent field and offer implications for professional development.

Confidentiality

The information in the study records will be kept confidential to the full extent allowed by law. Data will be stored securely by the principle investigator. No reference will be made in oral or written reports which could link you to the study. You will not be asked to write your name on any study materials so that no one can match your identity to the answers that you provide.

Compensation

For participating in this study you will receive a $25 Visa gift card at the completion of data collection. If you withdraw from the study prior to its completion, you will not receive any compensation and all data that has been collected up to that point will be destroyed.

What if you have questions about this study?

If you have questions at any time about the study or the procedures, you may contact, Tracy Foote White at Poe 502A Campus Box 7801 Raleigh, NC 27695, or tsfoote@ncsu.edu.

What if you have questions about your rights as a research participant?

If you feel you have not been treated according to the descriptions in this form, or your rights as a participant in research have been violated during the course of this project, you may contact Deb Paxton, Regulatory Compliance Administrator, Box 7514, NCSU Campus (919/515-4514).

Consent to Participate

"I have read and understand the above information. I have received a copy of this form. I agree to participate in this study with the understanding that I may choose not to participate or to stop participating at any time without penalty or loss of benefits to which I am otherwise entitled."

Indicate **YES** or **NO:**

- I give consent to be audiotaped during this study.
  ___Yes ___No
- I give consent to be videotaped during this study:
  ___Yes ___No
- I give consent for video data resulting from this study to be used for presentations on the study's findings.
  ___Yes ___No

Subject's signature____ Date __________________

Investigator's signature_____________________________________ Date __________________

Student Consent Form

Dear Undergraduate Calculus I Student:

You are currently enrolled in Dr./Instructor _________________’s Calculus I course. She/he has decided to participate in a research study that requires classroom observations. The observations will be used to collect evidence of all of the talk during class discussions. The discussions about mathematics will be analyzed. All classroom observations will be video-recorded. The video recordings will not be used to judge you in any way. The camera will be placed in the least intrusive position, as to not interfere with what is going on the class. The videos will be uploaded to the researcher’s password protected personal computer, and backed up on an external hard drive that will be stored in a locked location. The videos will be used to analyze the classroom discourse. If provided your consent, video excerpts may be used during presentations on the study’s findings.

Consent to be participate:
“I have read and understand the above information. I have received a copy of this form. I agree to participate in this study with the understanding that I may choose not to be video recorded or to stop being video recorded at any time without penalty.”

Indicate **YES** or **NO:**

- I give consent to be videotaped during this study:
  ___Yes ___No
- I give consent for video data resulting from this study to be used for presentations on the study’s findings.
  ___Yes ___No

*If you answered NO to the previous question, answer the following:

- I give consent for the audio from the video data resulting from this study to be used for presentations on the study’s findings.
  ___Yes ___No

In the event that you decide that you do not want to be video recorded you will be placed such that you are not in the view of the camera.

If you have any questions, please feel free to contact Tracy Foote White at Poe 502A Campus Box 7801 Raleigh, NC 27695, or tsfoote@ncsu.edu.

Print your name: ____ Date: ____________

Sign your name: ____ Date: ____________

Appendix E: Observation Protocol

Instructor:
Setting:
Date:
Class Time:
Number of Students:

Time	Description	Direct Quotes	*Reflective Notes/Thoughts*