ANISOTROPIC THEORY AND THE MEASUREMENT AND USE OF THE ANISOTROPIC FACTORS FOR ZIRCALOY-4 FUEL SHEATHS

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SUMMARY

Theories of the anisotropic mechanical behaviour of materials were examined and the applicability of the simplified theory of Hill to predict the mechanical behaviour of Zircaloy-4 nuclear fuel sheaths is discussed. Values of the three tensile anisotropic factors required by Hill's theory were obtained from three simple mechanical tests: longitudinal tensile, free end burst, and closed end burst tests. These values were found to depend on the metallurgical condition of the tube and on the amount of strain. For this latter reason, the Knoop microhardness method of Wheeler and Ireland was shown to be unsatisfactory.

To check the applicability of Hill's theory to Zircaloy-4 nuclear fuel sheathing tests were done on tubes in the as-received condition (cold-worked plus stress relieved at about 520°C) by internally pressurizing and then heating them at a constant rate of 25°C s^{-1} to a predetermined temperature, at which point the pressure was vented. Two sets of tests were done. In the first set, the ends were restrained from shortening because of the diametral expansion, but free to move with thermal expansion. From the results of this set of tests, the behaviour of unrestrained pressurized tubes was predicted. A second set of tests was done to check the predictions. The measured strains agreed well with the predictions up to a temperature of 660°C. Tests at 700°C and 750°C did not. The results at 700°C and 750°C could be predicted if one assumed the anisotropic factors appropriate to fully annealed material, even though the time spent above the previous stress relieving temperature was only 8 to 10 seconds. Electron microscopy showed that by 700°C the material had fully recrystallized, thus justifying the conclusion drawn from the mechanical tests.

This work shows that under tensile loading conditions, provided the metallurgical condition and hence the appropriate anisotropic factors are known, Hill's anisotropic theory of mechanical behaviour can be applied to Zircaloy-4 fuel sheaths. The anisotropic factors can be measured at room temperature using simple tests and applied to elevated temperature behaviour. The values of the anisotropic factors vary with metallurgical condition and plastic strain.

1. Introduction

Nuclear fuel sheaths are subject to a complex system of loads. Analysis of the stresses and strains caused by these loads is made difficult because the mechanical properties of α-zirconium alloys are anisotropic.

Rigorous anisotropic theory can appear quite formidable. As a result most engineers ignore it. The purpose of this paper is to show that for the tubular geometry of a nuclear fuel sheath simplifications can be made to reduce the equations to variations of the Huber-von Mises [1] and Lévy-von Mises [2,3] equations as proposed by Hill [4].

CANDU* fuel elements are closed end cylinders. When assessing the mechanical behaviour of fuel sheathing, one commonly uses the closed end burst test. This test, which provides a stress ratio (hoop/axial) of 2:1, results in zero change in length of an isotropic tube, all the diametral expansion being compensated by wall thinning. The strain ratio, $\varepsilon_z/\varepsilon_\theta$, was measured for many Zircaloy-4 tubes in test programs done by Hardy [5], Daniel [6] and the author. Over a temperature range from 600°C to 800°C it was found to be approximately -0.1. Therefore anisotropy must be taken into account.

The anisotropic parameters of fuel sheathing were measured and the results successfully applied to predict the mechanical behaviour of specimens under one set of loading conditions from results obtained under other conditions.

2. Anisotropic Theory

A theory of yielding and plastic flow of anisotropic metals was first proposed by Hill [4]. This theory was designed specifically to describe the behaviour of cubic metals where anisotropy was developed by fabrication processes. To simplify the equations, without affecting the arguments, we considered loading only in the principal directions. Terms involving the shear stresses are therefore zero.

Hill proposed three tensile anisotropic parameters which he designated as F, G and H. In r, θ, z cartesian co-ordinates the Huber-von Mises equivalent stress becomes

$$\bar{\sigma} = \left[F\,(\sigma_\theta - \sigma_z)^2 + G\,(\sigma_z - \sigma_r)^2 + H\,(\sigma_r - \sigma_\theta)^2\right]^{\frac{1}{2}} \qquad (1)$$

Yielding occurs when $\bar{\sigma} = Y$, the yield function, which for isotropic materials is the same as the yield stress in simple tension.

The Levy-von Mises equations become

$$d\varepsilon_r = d\lambda\,[H\,(\sigma_r - \sigma_\theta) + G\,(\sigma_r - \sigma_z)]$$

$$d\varepsilon_\theta = d\lambda\,[F\,(\sigma_\theta - \sigma_z) + H\,(\sigma_\theta - \sigma_r)] \qquad (2)$$

$$d\varepsilon_z = d\lambda\,[G\,(\sigma_z - \sigma_r) + F\,(\sigma_z - \sigma_\theta)]$$

where $d\lambda$ is a constant of proportionality which is defined by the following equation. The equivalent or combined strain increment, $\overline{d\varepsilon}$, may be derived as

$$\overline{d\varepsilon} = \frac{\left[H\,(Fd\varepsilon_r - Gd\varepsilon_\theta)^2 + F\,(Gd\varepsilon_\theta - Hd\varepsilon_z)^2 + G\,(Hd\varepsilon_z - Fd\varepsilon_r)^2\right]^{\frac{1}{2}}}{[HF + FG + GH]} = \bar{\sigma}\,d\lambda \qquad (3)$$

(see for example reference 7). If the material is isotropic, F = G = H = ½ and equations (1), (2) and (3) reduce to the more familiar forms.

* CANDU = Canada Deuterium Uranium

Because of the assumptions required by Hill's theory, Picklesimer and Rittenhouse [8,9] suggested that it may not apply to hexagonal metals, in particular zirconium alloys. Mannan and Rodriguez [10] showed a difference of 32% between the tensile and compressive yield strength of a Zircaloy-2 slab. Bogue [11], among others, has therefore developed a generalized anisotropic theory. His equation contained ten terms, the first three of which contributed only 2 or 3% to the final result for the most anisotropic Zircaloy-2 which he examined. The equation could therefore be reduced to seven terms with little introduction of error. These seven terms were the same as Hill's with the addition of one more to account for the Bauschinger effect, or the difference in tensile and compressive yield strength. Bogue solved the equations for the seven constants by trial and error using a digital computer.

For most of the tubes of interest in this investigation the tensile and compressive yield strengths in each co-ordinate direction were approximately equal. It was therefore decided to ignore the Bauschinger effect term and try to fit the experimental data to Hill's equation which requires only six constants. By restricting the experiments to tubes loaded so that the principal stresses are in the directions of the co-ordinate axes, the three shear stress terms are zero. Thus, only three constants required evaluation.

3. <u>Consequences of the Theory on Test Procedures</u>

Open end burst tests, where the pressurizing fluid is sealed by O-rings at the tube ends, and longitudinal tensile tests give uniaxial stressing in the θ and z directions respectively. The closed end burst test, which gives a stress ratio $\sigma_\theta/\sigma_z = 2/1$ applies stress in both directions simultaneously. The resulting equations are:

Uniaxial tensile test, $\sigma_r = \sigma_\theta = 0$

$$\left(\frac{d\varepsilon_\theta}{d\varepsilon_z}\right) = -\frac{F}{G + F} \tag{4}$$

Open end burst test, $\sigma_r = \sigma_z = 0$

$$\left(\frac{d\varepsilon_\theta}{d\varepsilon_z}\right) = -\frac{F + H}{H} \tag{5}$$

Closed end burst test, $\sigma_r = 0$, $\sigma_\theta = 2\,\sigma_z$

$$\left(\frac{d\varepsilon_\theta}{d\varepsilon_z}\right) = \frac{F + 2H}{G - F} \tag{6}$$

As only two of these equations are independent, a third independent equation is needed to solve for the three unknowns, F, G, and H.

Wheeler and Ireland [12] suggested that the geometric properties of the Knoop hardness indenter may be used to measure the shape of the yield surface, and hence the anisotropic factors. Six microhardness measurements are required which may be calibrated by measuring the yield strength in any one direction. Solving equation (1) for the uniaxial yield condition gives

$$F + G = \left(\frac{Y}{\sigma_z}\right)^2 \; ; \quad F + H = \left(\frac{Y}{\sigma_\theta}\right)^2 \; ; \quad G + H = \left(\frac{Y}{\sigma_r}\right)^2 \tag{7}$$

For isotropic materials, the yield function, Y equals the yield stress in simple tension.

When the yield stress is different in each of the co-ordinate directions, as in anisotropic materials, the definition of Y is no longer obvious. Y may be eliminated by dividing one of the equations (7) by the other two in turn. This leaves two equations with three unknowns. C 2/9

Unfortunately, these equations are not independent from those obtained by the strain measurements (equations 4, 5 and 6) so that the problem of only two equations with three unknowns remains.

In practice, the factors, F, G and H always appear as ratios in any mechanical test. Thus one may choose the third condition arbitrarily. Backofen et al. [13] suggest that only two independent anisotropic parameters exist. This approach was used also by Busby and Salvaggio [14] and by Duncombe [15]. Using Hill's notation and the order chosen in equations (1) and (2), Dumcombe's work gives

$$F + G = 1 \tag{8}$$

as a third condition. Ross-Ross et al. [16] suggested that since for isotropic materials $F = G = H = 0.5$, the third condition should be

$$F + G + H = 1.5 \tag{9}$$

Both equations (8) and (9) are correct with isotropic materials but they lead to different values of anisotropic factors. However, both equations give identical predictions of the behaviour. Pending a satisfactory definition of the yield function, Y, for anisotropic materials, the use of either equation is valid provided consistency is maintained. When comparing results between different authors it is also important to check on the order in which F, G and H are selected relative to the coordinate axes.

Bogue [11] points out that the measurement of strain ratios provides a far more sensitive measure of anisotropic behaviour than does measurement of the yield stresses. Strains can be measured accurately while the yield stress, particularly for materials without a sharp yield point, is difficult to define. In addition, equations (7) show that in calculating the anisotropic factors, the yield stress must be squared, also squaring any measurement error. As a result, the present work has been based on calculating the Hill factors from strain measurements using the Lévy-von Mises equations.

4. Experimental Procedure

Several batches of Zircaloy-4 tube, of similar fabrication history, were tested in three metallurgical conditions:

(1) as-received (cold worked plus unspecified stress relief),

(2) stress relieved for four hours at 525°C (550°C for one batch),

(3) annealed one hour at 800°C followed by furnace cooling.

One batch of a different fabrication history was tested in the as-received condition.

Strain ratio measurements were done on specimens tested at room temperature under uniaxial tensile, free end burst and closed end burst test conditions. In each test a single specimen was strained to yield, unloaded and measured, reloaded until an additional small amount of yielding was observed and again unloaded and measured. This process was repeated to provide from four to seven points from each specimen. Average strain ratios were taken from the slopes of the resulting lines. Values of F, G and H were calculated using equations (4), (5) and (6) and one of equations (8) or (9). Normally only two of the above tests were done. When all three were done, three sets of values of F, G and H were calculated and the results averaged.

Knoop hardness measurements were made on a few specimens to ascertain the shape of the yield surface and to compare the values of F, G and H obtained by the two methods.

To check the validity of the results, tests were done using the apparatus described by Hardy [5]. This consisted of a vacuum chamber in which tubes were heated by electrical resistance heating. A constant internal pressure was maintained by inert gas. An hydraulic system was added to provide an axial load to prevent shortening. Tests were done both with the ends restrained and with the ends free. Full length (500 mm) fuel sheaths were heated at a rate of 25°C s^{-1} while under internal pressure. On reaching a pre-determined temperature the heating was stopped, the internal pressure vented and the axial restraining load released. The diametral strain was measured by a micrometer at 25 mm intervals along the test section on two orthogonal planes and the results averaged.

5. Results

Figures 1 and 2 give examples of the type of flow surface obtained by Knoop hardness measurements. Figure 1 shows differing values of Knoop hardness number, and hence yield stress, in the three co-ordinate directions but similar values in tension and compression along any one axis. Figure 2 shows higher compressive yield values in the r and θ directions. Thus the assumption required by Hill's theory of equal tensile and compressive yield strengths is not necessarily met.

Figures 3, 4 and 5 show the results of the strain ratio measurements. In general, straight lines could be drawn through the data points up to values of about 0.03 true strain. These lines did not always extrapolate to zero strain. The values calculated for F, G and H using both equation (8) and equation (9) are shown on Table I. Batches MLI-3034, -3804 and -7141 had similar fabrication histories and textures: tube reduced 60%, stress relieved, texture coefficient ratio A/C* = 0.2 to 0.5 [17,18]. Similar metallurgical conditions gave similar values of F, G and H and the values changed progressively from the as-received condition to the fully annealed condition. MLI-788 was cold drawn 16 passes with intermediate anneals resulting in a texture coefficient ratio A/C = 1.72 [17,18].

The results of the temperature ramp tests on the internally pressurized 500 mm long fuel sheaths are shown on Figure 6. A series of tests was done, one test for each point shown, with longitudinal shortening prevented. The solid curve is drawn through the results. The scatter shown is typical for this type of test [5]. Using the measured values of F, G and H for as-received tube, unrestrained end tests were predicted to give 29% more diametral strain. This prediction is shown as the dashed curve on Figure 6. A similar calculation using the stress relieved values of F, G and H predicted 41% more diametral strain than the restrained end test, as shown by the dotted curve on the figure. At peak temperatures below 670°C the unrestrained end test results agree well with the predicted curve assuming as-received values of F, G and H. All three points give an increase in diametral strain of about 30%. At 700°C and 755°C the increase in diametral strain was about 50% implying that fully annealed values of F, G and H would be more appropriate. Electron microscopy of several specimens showed that over the interval from about 660°C to 700°C the material recrystallized fully.

* An A texture represents tangential orientation of basal <0001> poles and a C texture represents radial basal poles.

6. Discussion

Although Hill's theory has limitations it provides a simple means of predicting strains under multiaxial loading. To successfully apply it errors introduced by violation of the assumptions must be small.

The strain measurements in the present work, as shown on Figures 3, 4 and 5 did not necessarily extrapolate to zero true strain. Since the data were self-consistent and repeatable, this is probably a real effect suggesting that over about the first 0.1% true plastic strain the anisotropic behaviour changes. In addition, beyond about 5% true plastic strain the strain ratio measurements again become non-linear. Thus, measurements based on yield at zero plastic strain and those based on intermediate or large plastic strains will all give different results. To illustrate this, a "flow surface" calculated from the strain measurements obtained at intermediate true plastic strains was compared with that obtained from Knoop microhardness measurements on the same material. Knoop microhardness measurements typically produce strains in the range of seven to ten percent. In Table II, the ratios of "yield" stress calculated by the two methods are shown with the value in the z-direction being arbitrarily set at unity. The maximum percent difference in the values obtained was 17.7%, in the radial direction. One is therefore faced with the result that for the Zircaloy-4 tubes tested in the present work, F, G and H are strain dependent. Maki et al. [19] show similar curvature over the first portion of the strain ratio curves and Dressler et al. [20] show similar differences between yield surfaces obtained by different methods.

On applying the measured values of F, G and H to predict sheath behaviour, Figure 6 showed the discrepancy for the two points at 700°C and 755°C could be resolved provided one used values appropriate to annealed material. Metallographic examination showed that these two specimens were fully recrystallized, justifying the conclusion. Therefore, provided the metallurgical condition and hence appropriate values of F, G and H are known, Hill's equation provides a prediction of the behaviour to within experimental error, about 5%.

For the tube reduced material used in this investigation, the contribution of Bogue's Bauschinger effect term must be small. This is not surprising in view of the results of Figure 1 which show that the yield strength along each co-ordinate axis was in fact similar in tension and compression.

Unfortunately there was not sufficient material available from batch MLI-788 to test in the manner shown on Figure 6. In this batch, Figure 2 shows significant differences between the tensile and compressive yield strengths. For material of this type, agreement with Hill's theory may depend on the testing procedure. Both in the tests shown in Figure 6 and the ones used to determine the anisotropic factors, the principal stresses were all zero or tensile. It is probable that if measurements on MLI-788 had been done in compression, a different set of anisotropic constants would have been found.

Incorrect predictions may be expected where the anisotropic constants are measured in the tensile regime and applied in the compressive or mixed regime. For thin wall tubes where the major plastic strains are expected to be tensile, this possibility may be safely ignored. For components made from thick sheet, such as those tested by Rittenhouse and Picklesimer, where both tensile and compressive stresses may be involved, asymmetry in the yield surface may result in unacceptable errors if the Bauschinger effect term is neglected.

Therefore, although the methods described above have proved to be very effective for describing the tensile behaviour of thin wall Zircaloy-4 fuel sheaths, the same may not be true for other components.

7. Conclusions

Hill's anisotropic theory can be applied to thin wall tubing but large differences between tensile and compressive yield strengths may introduce errors. The values of the anisotropic factors depend both on the metallurgical condition and on the strain. As a result the Knoop microhardness method should only be used to obtain the approximate shape of the flow surface.

8. Acknowledgements

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TABLE I

Anisotropic factors showing the variations due to metallurgical condition and crystallographic texture.

Eq'n	(8)	(9)	(8)	(9)	(8)	(9)
MLI	3804 As Received		3804 4 h at 525°C		3804 1 h at 800°C	
F	0.556	0.738	0.673	0.865	0.759	0.956
G	0.444	0.588	0.327	0.419	0.241	0.304
H	0.131	0.174	0.168	0.216	0.190	0.240
MLI	3034 As Received		3034 4 h at 525°C		3034 1 h at 800°C	
F	0.644	0.773	0.714	0.934	0.743	0.956
G	0.356	0.532	0.287	0.374	0.257	0.331
H	0.162	0.195	0.147	0.192	0.165	0.213
MLI	7141 As Received		7141 4 h at 550°C		788* As Received	
F	0.702	0.829	0.717	0.916	0.375	0.416
G	0.298	0.354	0.283	0.362	0.625	0.692
H	0.268	0.317	0.176	0.222	0.353	0.392

* MLI-788 tube had different fabrication history and crystallographic texture to the other batches.

TABLE II

Comparison of flow surface from Knoop Hardness measurements with the yield surface (greater than 0.2% strain) obtained from strain measurements. Values are in arbitrary units normalized to give $\sigma_z = 1$.

	Knoop Hardness Method	Strain Measurement Method	% Difference
σ_z	1.0	1.0	0
σ_θ	1.086	1.059	- 2.5
σ_r	1.257	1.479	+17.7

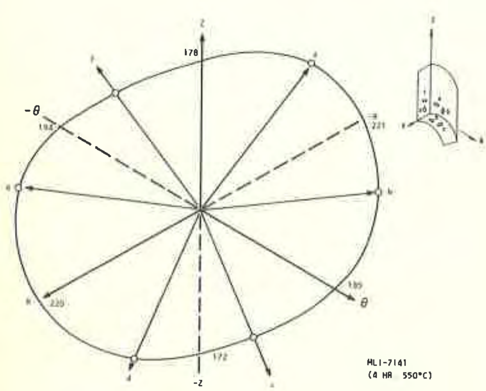

FIGURE 1

Knoop hardness number yield surface for typical fuel sheath.

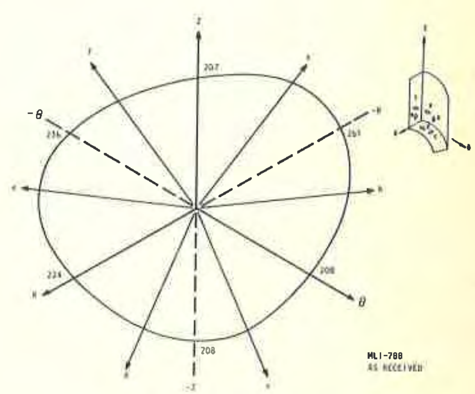

FIGURE 2

Knoop hardness number yield surface for fuel sheath with unusual crystallographic texture.

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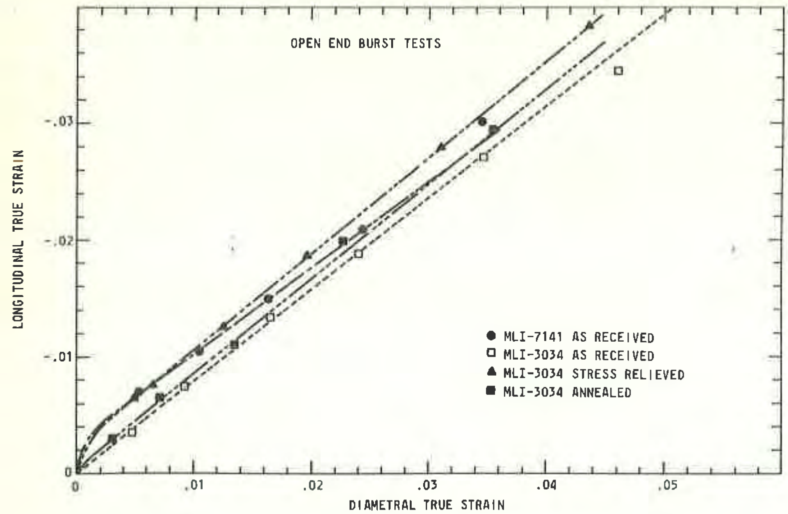

FIGURE 3A

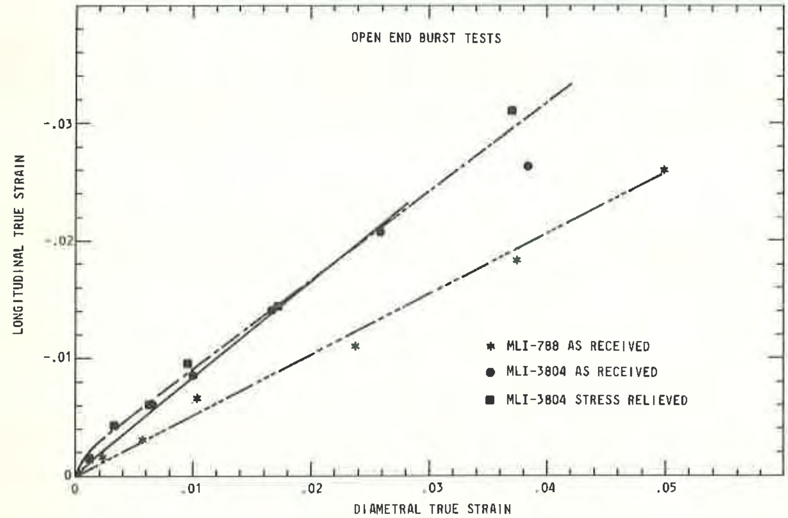

FIGURE 3B

FIGURE 3A, B Longitudinal vs. Diametral true plastic strain for open end burst tests.

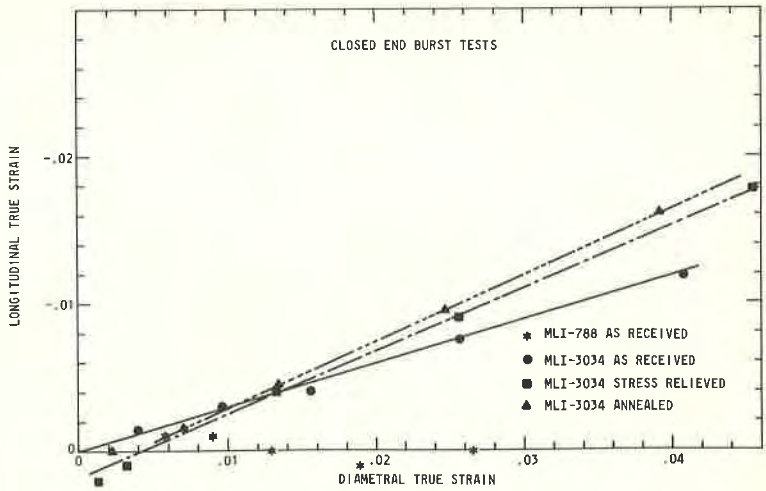

FIGURE 4A

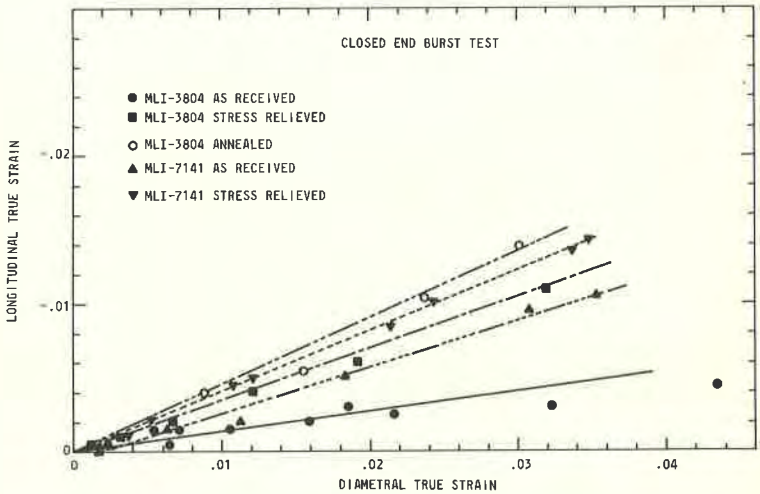

FIGURE 4B

FIGURE 4A, B Longitudinal vs. Diametral true plastic strain for closed end burst tests.

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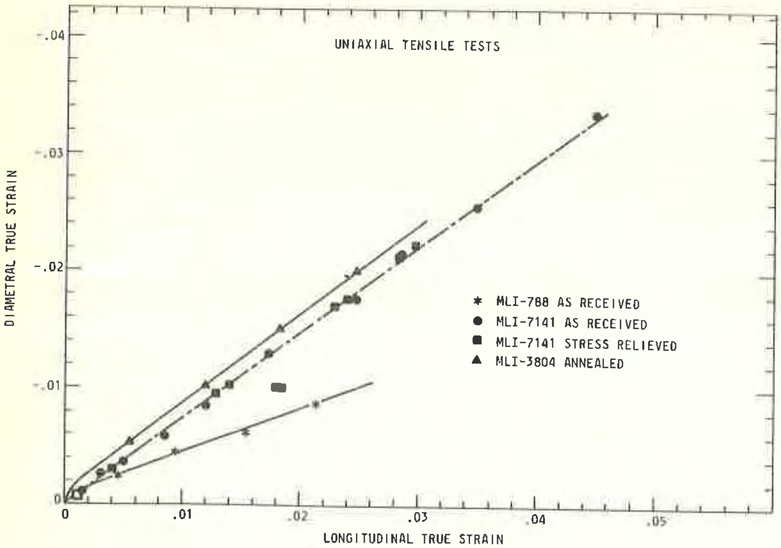

FIGURE 5 Diametral vs. Longitudinal true plastic strain for uniaxial tensile tests.

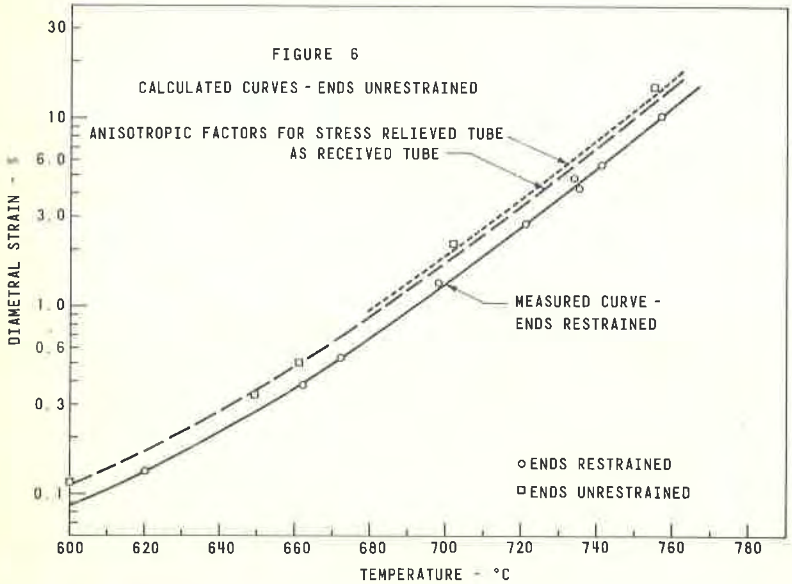

FIGURE 6 Comparison of measured and predicted diametral plastic strain for tubes with and without ends restrained.